45. *The Structure of Bebutov Dynamical System*

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1. Introduction. Let X be a metric space. A flow or a dynamical system on X is defined to be the triple (X, R, π) consisting of X, the real line R and a map $\pi : X \times R \to X$ such that

a) $\pi(x, 0) = x$, $x \in X$,

b) $\pi(\pi(x, s), t) = \pi(x, s+t)$, $s, t \in R$, $x \in X$,

c) π is continuous on $X \times R$.

Given a dynamical system on X, the space X is called the phase space of the dynamical system.

Let X_u be the set of all complex-valued continuous functions on R. X_u becomes a metric space with the metric

$$\rho(\varphi, \psi) = \sup_{T>0} \min \left\{ \max_{|x| \leq T} |\varphi(x) - \psi(x)|, \frac{1}{T} \right\}.$$

Define a map

$$f_u : X_u \times R \longrightarrow X_u$$

by

$$f_u(\varphi, t) = \varphi \circ g_t, \qquad \varphi \in X, \ t \in R,$$

where $g_t(x) = x + t$ for any $x \in R$. Then a dynamical system (X_u, R, f_u) is obtained, which is called the Bebutov dynamical system [1]. The Bebutov dynamical system is important in the sense that a large class of compact flows (i.e., the flows such that the phase spaces are compact) may be embedded in it by virtue of the theorem of Bebutov-Kakutani [2]: a necessary and sufficient condition for a compact flow to be isomorphic to some subsystem of the Bebutov dynamical system is that its set of rest points be homeomorphic to some subset of the real line R.

The purpose of this paper is to study the structure of the phase spaces of the Bebutov dynamical system and its compact subsystem.

The results obtained are:

(a) any orbit which is dense in X_u is positively or negatively Poisson stable (Theorem 3.1),

(b) there exists an orbit which is dense in X_u, positively or negatively Poisson stable, and neither positively nor negatively receding (Theorem 3.2),

(c) the phase space of the compact subsystem of the Bebutov dynamical system (X_u, R, f_u) is a border set in X_u (Theorem 4.1).

2. Definitions and notations. The sets