5. *An Approach to Linear Hyperbolic Evolution Equations by the Yosida Approximation Method*

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(Communicated by Kôsaku YOSIDA, M. J. A., Jan. 12, 1978)

§ 1. **Introduction.** T. Kato [1, 2] studied the Cauchy problem for linear "hyperbolic" evolution equations in a general Banach space X:

(1.1) $\quad (du/dt)+A(t)u(t)=0, \quad u(s)=x, \quad 0\le s\le t\le T<\infty,$

where $-A(t)$ is the generator of a (C_0)-semigroup in X for each t. He proved the basic existence theorem [1; Theorem 4.1] by the Cauchy's method analogous to ordinary differential equations. He posed a question whether it is possible or not to prove the theorem by the Yosida approximation method. In this paper we will answer the question affirmatively under the assumptions of Kato [1; Theorem 4.1]. In § 2 we treat the "stable" case about the family $\{A(t)\}$; we study some properties of the Yosida approximation, then in § 3 we prove the existence theorem. Finally in § 4 we give some comments how our arguments are modified in the case of "quasi-stability" [2].

§ 2. **Theorem.** We follow Kato [1] in notation and terminology. Let X and Y be real Banach spaces with Y densely and continuously embedded in X. We assume that $-A(t)$ is the generator of a (C_0)-semigroup on X. Further assume

(i) $\{A(t)\}$ is stable; i.e., there are constants M, β such that:

$$\|(A(t_k)+\lambda)^{-1}\cdots(A(t_1)+\lambda)^{-1}\|\le M\cdot(\lambda-\beta)^{-k}$$

for $\lambda>\beta$ and $0\le t_1\le\cdots\le t_k\le T$, $k=1,2,\cdots$.

(ii) Y is $A(t)$-admissible for each t; that is, the semigroup generated by $-A(t)$ leaves Y invariant and forms a (C_0)-semigroup on Y. And if $\tilde{A}(t)$ is the part of $A(t)$ in Y, then $\{\tilde{A}(t)\}$ is stable with some constants $\tilde{M}, \tilde{\beta}$[1, p. 242].

(iii) $Y\subset D(A(t))$ for each t and $A(t)$ is norm continuous from $[0, T]$ into $B(Y, X)$.

Hereafter we assume $\beta, \tilde{\beta}>0$ for simplicity.

A family $\{U(t,s)\,;\,0\le s\le t\le T\}$ is called the evolution operator for $\{A(t)\}$ if it satisfies the following conditions:

(a) $U(t,s)$ is strongly continuous (X) in s, t and, $U(t,t)=I$ and $\|U(t,s)\|\le M\cdot\exp[\beta(t-s)]$.

(b) $U(t,r)=U(t,s)U(s,r)$, $r\le s\le t$.

(c) $(\partial/\partial t)^{+}U(t,s)y|_{t=s}=-A(s)y$ for $y\in Y$, $0\le s<T$.

(d) $(\partial/\partial s)U(t,s)y=U(t,s)A(s)y$ for $y\in Y$, $0\le s\le t\le T$.