62. *The Fourier Transform of the Schwartz Space on a Symmetric Space*

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1. Introduction. Let S be a symmetric space of the noncompact type and let $L^2(S)$ denote the space of square integrable functions on S with respect to the invariant measure. In his paper [7], S. Helgason characterized the image of $L^2(S)$ by the Fourier transform.

The purpose of this paper is to give a characterization of the image of the Harish-Chandra's Schwartz space by the Fourier transform. As an immediate consequence we obtain the above mentioned result of S. Helgason (the characterization of the image of $L^2(S)$ by the Fourier transform). The proofs of the results are given in [2].

2. Notation and preliminaries. If M is a manifold (satisfying the second countability axiom), following Schwartz $\mathcal{D}(M)$ denotes the the space of C^∞ functions on M with compact support. If V is a real vector space $\mathcal{S}(V)$ denotes the space of rapidly decreasing functions on V (see [8]) and $\boldsymbol{D}(V)$ denotes the algebra of differential operators with constant coefficients on V.

If G is a Lie group and H a closed subgroup, G/H denotes the space of left cosets gH, $g \in G$. $\boldsymbol{D}(G/H)$ denotes the algebra of differential operators on homogeneous space G/H which are invariant under left translations by G. We write $\boldsymbol{D}(G)$ for $\boldsymbol{D}(G/e)$, where e is the identity of G.

Let S be a symmetric space of the noncompact type that is a coset space $S=G/K$ where G is a connected semisimple Lie group with finite center and K a maximal compact subgroup. Let $\mathfrak{g}$ and $\mathfrak{k}$ denote the Lie algebras of G and K respectively. Let B be the Killing form of $\mathfrak{g}$ and θ the Cartan involution which associates with the Cartan decomposition $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$. Let $\mathfrak{a}\subset\mathfrak{p}$ be a maximal abelian subspace and $\mathfrak{a}^*$ its dual. Put $A=\exp\mathfrak{a}$. For $\lambda\in\mathfrak{a}^*$ put

$$\mathfrak{g}_\lambda=\{X\in\mathfrak{g}\mid[H,X]=\lambda(H)X,\ \text{for all } H\in\mathfrak{a}\}.$$

If $\lambda\neq 0$ and $\mathfrak{g}_\lambda\neq\{0\}$ then λ is called a restricted root and $m_\lambda=\dim(\mathfrak{g}_\lambda)$ is called its multiplicity. Let $\mathfrak{g}_c$ and $\mathfrak{a}_c^*$ denote the complexifications of $\mathfrak{g}$ and $\mathfrak{a}^*$ respectively. If λ, $\mu\in\mathfrak{a}_c^*$ let $H_\lambda\in\mathfrak{a}_c$ (the complex subspace of $\mathfrak{g}_c$ spanned by $\mathfrak{a}$) be determined by $\lambda(H)=\langle H_\lambda, H\rangle$ for $H\in\mathfrak{a}$ and put $\langle\lambda,\mu\rangle=\langle H_\lambda, H_\mu\rangle$. Since B is positive definite on $\mathfrak{p}$ we put $\|\lambda\|=\langle\lambda,\lambda\rangle^{1/2}$