59. *The Behavior of Solutions of the Equation of Kolmogorov-Petrovsky-Piskunov*

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1. Given a source function $F(u)$ on $[0,1]$ which is positive on $0<u<1$ with $F(0)=F(1)=0$, continuously differentiable on $0\leqq u\leqq 1$ and $F'(0)>0$, let us consider the Cauchy problem

$$(1)\qquad \frac{\partial u}{\partial t}=\frac{1}{2}\frac{\partial^2 u}{\partial x^2}+F(u) \qquad t>0,\ x\in R=(\infty,+\infty)$$
$$\lim_{t\downarrow 0} u(t,x)=f(x),$$

where an initial function f is piecewise continuous on R with $0\leqq f\leqq 1$.

Let w_c denote a propagating front associated with speed c: $w_c(x-ct)$ is a non-trivial solution of (1)[*)] ($0\leqq w_c\leqq 1$), with normalization $w_c(0)=1/2$. Our interest in this article lies in such phenomena that

(2) $\quad u(t,x+m(t))$ *converges to* $w_c(x)$ *as* $t\to\infty$,

where

$$m(t)=\sup\left\{x\,;\,u(t,x)=\frac{1}{2}\right\}.$$

If $f\not\equiv 0$ and $f(x)\to 0$ as $x\to\infty$, we have that $u(t,x)\to 0$ as $x\to\infty$ and $u(t,x)\to 1$ as $t\to\infty$ (cf. [1]) and in particular that $m(t)$ has a definite value for large t. Aronson and Weinberger proved in [1] that there is a positive constant c_0 called the minimal speed such that the propagating front associated with speed c exists iff $c^2\geqq c_0^2$ ($c_0^2\geqq 2F'(0)$; the propagating front is unique up to the translation for each c) (cf. also [3]). Such a phenomenon as described in (2) was first observed by Kolmogorov, Petrovsky and Piskunov [3]: they proved that (3) holds with $c=c_0$ if we take $f=I_{(-\infty,0)}$ (the indicator function of the negative real axis). Kametaka [2] proved (2) when f belongs to a certain class of monotone functions. These are improved in the theorems of the next section which confirm that (2) is valid to a wide class of initial functions that contains all $f(0\leqq f\leqq 1)$ with non-empty compact support.

2. Let $A(x)$ be a positive function on R such that $A(x+x_0)\sim A(x)$ as $x\to\infty$ for each $x_0\in R$. We will assume one of the following conditions on the behavior of f for large positive x:

(3) $\quad f(x)=0 \quad$ for $x>N_1(N_1\in R)$ and $f\not\equiv 0$

or

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*) Trivial solutions are $u\equiv 0$ and $u\equiv 1$.