57. On the 2-Components of the Unstable Homotopy Groups of Spheres. II

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This note is the continuation of the part I with the same title. We will state the results on the 2-components of the unstable homotopy groups of spheres for the following cases: π^n_{n+29} and π^n_{n+30} for all n*); π^n_{n+31} for n*)$\leqq 29$. Moreover, the following groups will be given: π^n_{n+32} and π^n_{n+33} for n*)$\leqq 8$. But the group π^9_{40} is not determined completely and the group extensions are not settled for π^{10}_{41} and π^n_{n+33} for $n=6, 7$ and 8.

5. On the 29-stem. *There are following new elements*: $\bar{\varepsilon}'$, $\delta' \in \pi^6_{35}$ *and* $\delta'' \in \pi^7_{36}$ *with the Hopf invariants* $\pm\bar{\varepsilon}_{11}$ (*mod other elements*), δ_{11} (*mod* $\bar{\mu}_{11}\circ\sigma_{28}$), *and* ϕ_{13} (*mod* $4\nu_{13}\circ\bar{\kappa}_{16}$) *respectively*.

$$\pi^3_{32}=Z_2\{\bar{\alpha}\circ\nu^2_{26}\}\oplus Z_2\{\nu'\circ\eta_6\circ\mu_{3,7}\}\oplus Z_2\{\eta_3\circ\varepsilon_4\circ\bar{\kappa}_{12}\},$$

$$\pi^5_{34}=Z_2\{\phi_5\circ\nu^2_{28}\}\oplus Z_2\{\nu_5\circ\bar{\kappa}_8\circ\nu^2_{28}\}\oplus Z_2\{\nu_5\circ\bar{\sigma}_8\circ\sigma_{27}\}\oplus Z_2\{\nu^3_5\circ\bar{\kappa}_{14}\}\oplus Z_2\{\nu_5\circ\eta_8\circ\mu_{3,9}\}\oplus Z_2\{\eta_5\circ\varepsilon'_6\circ\bar{\kappa}_{14}\}.$$

In the above group, the following relation holds: $\phi_5\circ\nu^2_{28}\equiv\nu_5\circ\sigma_8\circ\bar{\sigma}_{15}$ (*mod* $\nu_5\circ\bar{\kappa}_8\circ\nu^2_{28}-\nu^2_5\circ\bar{\kappa}_{11}\circ\nu_{31}$).

Now we define elements by Toda brackets: $\delta'\in\{\sigma''\circ\sigma_{13},\sigma_{20},2\sigma_{27}\}_3$, $\delta''\in\{\sigma'\circ\sigma_{14},\sigma_{21},2\sigma_{28}\}_4$. Then we have $2\delta''=-E\delta'$ *and* $E^2\delta''=2(\sigma_9\circ\sigma^*_{16})$. Moreover there are following important results: $\Delta(\bar{\varepsilon}_{13})=2\bar{\varepsilon}'$ *for some* $\bar{\varepsilon}'\in\pi^6_{35}$ *and* $2\delta'\equiv\nu^3_6\circ\bar{\kappa}_{15}=\nu_6\circ\bar{\kappa}_9\circ\nu^2_{29}$ (*mod* $\nu_6\circ\sigma_9\circ\bar{\sigma}_{16}$). Using these results, we have

$$\pi^6_{35}=Z_4\{\delta'\}\oplus Z_4\{\bar{\varepsilon}'\}\oplus Z_2\{\phi_6\circ\nu^2_{29}\}\oplus Z_2\{\eta_6\circ\varepsilon_7\circ\bar{\kappa}_{15}\},$$

$$\pi^7_{36}=Z_8\{\delta''\}\oplus Z_2\{\sigma'\circ\varepsilon_{14}\circ\kappa_{22}\}\oplus Z_2\{\sigma'\circ\omega_{14}\circ\nu^2_{30}\}\oplus Z_2\{\phi_7\circ\nu^2_{30}\}\oplus Z_2\{\eta_7\circ\varepsilon_8\circ\bar{\kappa}_{16}\}.$$

In the above group, we have $\sigma'\circ\omega_{14}\circ\nu^2_{30}\equiv E\bar{\varepsilon}'$ (*mod* $E^2\pi^5_{34}$). This is obtained showing that $\sigma'\circ\omega_{14}\circ\nu^2_{30}$ *is not double suspended*: If $\sigma'\circ\omega_{14}\circ\nu^2_{30}\in E^2\pi^5_{34}$, we may construct the Toda bracket $\{\sigma'\circ\omega_{14}+x\phi_7+y\nu_7\circ\bar{\kappa}_{10},\nu^2_{30},2\iota_{36}\}_1$ whose Hopf invariant is $\bar{\varepsilon}_{13}$ (*mod other elements*). *Then we see* $\Delta\pi^{13}_{37}=0$, *which contradicts the fact that* $H\Delta(\bar{\varepsilon}_{13})=2\bar{\varepsilon}_{11}\neq 0$.

$$\pi^9_{38}=Z_{16}\{\sigma_9\circ\sigma^*_{16}\}\oplus Z_2\{\sigma_9\circ\omega_{16}\circ\nu^2_{32}\}\oplus Z_2\{\sigma_9\circ\varepsilon_{16}\circ\kappa_{24}\}\oplus Z_2\{\sigma_9\circ\nu_{16}\circ\bar{\sigma}_{19}\}\oplus Z_2\{\eta_9\circ\varepsilon_{10}\circ\bar{\kappa}_{18}\}.$$

$$\pi^{10}_{39}=Z_8\{\Delta(\bar{\kappa}_{21})\}\oplus Z_2\{\Delta(EA_2)\}\oplus Z_{16}\{\sigma_{10}\circ\sigma^*_{17}\}\oplus Z_2\{\sigma_{10}\circ\nu_{17}\circ\bar{\sigma}_{20}\}.$$

This results from the relation $4\Delta(\bar{\kappa}_{21})=\sigma_{10}\circ\varepsilon_{17}\circ\kappa_{25}$.

We will use hereafter the metastable periodic elements: $\pi^{11}_{40}=Z_2\{C_1\circ\bar{\mu}_{23}\}\oplus Z_{16}\{\sigma_{11}\circ\sigma^*_{18}\}\oplus Z_2\{\sigma_{11}\circ\nu_{18}\circ\bar{\sigma}_{21}\}$, $\pi^{12}_{41}=Z_4\{\Delta(\nu^*_{25})+2\sigma_{12}\circ\sigma^*_{19}\}\oplus Z_2\{A_1\circ\bar{\mu}_{24}\}\oplus Z_2$

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*) We omit the cases that $n=2, 4$ and 8 (c.f. Proposition 4.4 of [11]).