56. *Cartan Connection and Characteristic Classes of Foliations*

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1. Introduction. In this note we define characteristic classes of conformal and projective foliations, keeping in mind the strong vanishing theorem of Nishikawa and Sato concerning the Pontrjagin classes of the normal bundles [5], and describe the relationship of them with those of smooth foliations defined by Bott and Haefliger [1] and those of Riemannian foliations due to Lazarov and Pasternack [4] (see also Kamber and Tondeur [2] for a general theory). The main point of the construction is the use of Cartan connection, which also yields new characteristic classes for Riemannian foliations.

2. Riemannian case. Let F be a codimension n Riemannian foliation on a smooth manifold M and let $O(F)$ be the orthonormal frame bundle of F. If we denote $\mathfrak{e}(n)$ for the Lie algebra of the group of Euclidean motions on $\boldsymbol{R}^n$, then the canonical form and the unique Riemannian connection form on $O(F)$ define a d.g.a. map

$$\phi: W(\mathfrak{e}(n)) \to \Omega^*(O(F))$$

where $W(\mathfrak{e}(n))$ is the Weil algebra of $\mathfrak{e}(n)$ and $\Omega^*(O(F))$ is the de Rham complex of $O(F)$. Now ϕ has a kernel I: the ideal generated by the facts (i) torsionfree-ness of the Riemannian connection (ii) the first Bianchi's identity and (iii) the curvature form is horizontal. Let $\tilde{W}(\mathfrak{e}(n)) = W(\mathfrak{e}(n))/I$ and assume that the normal bundle of F is trivialized by a cross section $s: M \to O(F)$, then we obtain

$$\phi^*: H^*(\tilde{W}(\mathfrak{e}(n))) \longrightarrow H^*_{DR}(O(F)) \xrightarrow{s^*} H^*_{DR}(M).$$

Since this construction is functorial, we have

$$\phi^*: H^*(\tilde{W}(\mathfrak{e}(n))) \to H^*(BR\bar{\Gamma}_n; \boldsymbol{R}),$$

where $BR\bar{\Gamma}_n$ is the classifying space for codimension n Riemannian Haefliger structures with trivial normal bundles. As for the cohomology of $\tilde{W}(\mathfrak{e}(n))$, we have

Theorem 1. $H^*(\tilde{W}(\mathfrak{e}(n))) = H^*(\tilde{W}(\mathfrak{so}(n))) + \sum_{\substack{0 \leq p < n \\ p:\text{even}}} r_p H^*(\mathfrak{so}(n)).$

Here $H^*(\tilde{W}(\mathfrak{so}(n)))$ is the cohomology of the truncated Weil algebra of $\mathfrak{so}(n)$ studied by Kamber and Tondeur [2] and is the same as the characteristic classes defined by Lazarov and Pasternack. $\phi^*(r_0)$ is the

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