39. A Generalization of Local Class Field Theory by Using K-groups. I

By Kazuya KATO*)

Department of Mathematics, Faculty of Science, University of Tokyo

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§0. **Introduction.** This note is a summary of our recent results on a generalization of local class field theory. Details will be published elsewhere.

Let F be a field which is complete with respect to a discrete valuation and with finite residue field. Let K be a field which is complete with respect to a discrete valuation and with residue field F. In this Part I, we shall study abelian extensions of K. The case in which F is a function field of one variable over a finite field and a generalization of our results will be studied in Part II ([1]).

§1. In Part I, let F denote a field which is complete with respect to a discrete valuation and with finite residue field, and let K denote a field which is complete with respect to a discrete valuation and with residue field F, and let K^{ab} denote the maximum abelian extension of K.

Theorem 1. (1) *There exists a canonical homomorphism*

$$\Phi : K_2(K) \longrightarrow \mathrm{Gal}(K^{ab}/K)$$

having the following property: For each finite abelian extension L of K, Φ induces an isomorphism

$$K_2(K)/N_{L/K}K_2(K) \cong \mathrm{Gal}(L/K),$$

where $N_{L/K}$ denotes the norm map in K_2-theory.

(2) *$L \mapsto N_{L/K}K_2(L)$ is a bijection from the set of all finite abelian extensions of K in a fixed algebraic closure of K to the set of all open subgroups of finite indices of $K_2(K)$ with respect to the topology defined later in* §4.

This is closely connected with the following result on the Brauer group of K.

Theorem 2. *There exists a canonical isomorphism*

$$\Psi : \mathrm{Br}(K) \xrightarrow{\sim} \mathrm{Hom}_c(K^*, \boldsymbol{Q}/\boldsymbol{Z})_{\mathrm{tor}}$$

having the following property, where K^* denotes the multiplicative group of K and $\mathrm{Hom}_c(K^*, \boldsymbol{Q}/\boldsymbol{Z})_{\mathrm{tor}}$ denotes the torsion part of the group of all continuous homomorphism $K^* \to \boldsymbol{Q}/\boldsymbol{Z}$ with respect to the topology

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