# 37. *On the Continuous Cohomology of the Lie Algebra of Vector Fields*

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§1. Let $M$ be a smooth manifold and $\mathcal{L}_M$ the topological Lie algebra of all smooth vector fields on $M$. Recently Haefliger ([4]) and G. Segal (unpublished) proved that the continuous cohomology $H(\mathcal{L}_M)$ of $\mathcal{L}_M$ with the trivial coefficient is isomorphic to the cohomology of the cross-section space of a canonically defined fibre bundle over $M$, as it was conjectured by Bott and Fuks.

As for the case associated with a non-trivial coefficient $\mathcal{A}$, Locik ([5][7]) has computed the cohomology of a certain subcomplex (called diagonal) of the standard cochain complex, and Reshetnikov ([9]) has announced partial results concerning the total continuous cohomology $H(\mathcal{L}_M, \mathcal{A})$. In this note, we state a theorem which reduces the calculation of $H(\mathcal{L}_M, \mathcal{A})$ to that of $H(\mathcal{L}_M)$ and the diagonal cohomology $H_\Delta(\mathcal{L}_M, \mathcal{A})$ modulo the computation of differential torsion product. Thus the problem is reduced to a purely algebraic one. Details will be published elsewhere.

After the author had obtained the results (Theorems 1–3), he heard from Professor Fuks that they had been obtained by him and G. Segal in 1974, although they had never published them.

The author would like to express his gratitude to Professor Fuks, who taught him the neat consequence Theorem 4 of Theorem 3 and remarked that the case of the topological Lie algebra $\mathcal{L}_M^c$ of vector fields with compact supports can be treated similarly. The author would like to thank Professors A. Tsuchiya and S. Morita for their valuable helps in proving Theorem 2.

Section 2 recalls briefly the definition of Lie algebra cohomology and states the main theorem (Theorem 1) in the case $\mathcal{A}=C^\infty(M)$, elements of $\mathcal{L}_M$ acting on $C^\infty(M)$ as derivations. We state a topological interpretation of $H(\mathcal{L}_M, C^\infty(M))$ in Section 3 and apply Theorem 1 to the particular cases $M=\boldsymbol{R}^n$ and $M=S^1$ in Section 4. Section 5 states the main theorem in the general case. We consider the case of $\mathcal{L}_M^c$ in Section 6.

§2. Let $\mathcal{W}$ be a topological $\mathcal{L}_M$-algebra. Let $C^p(\mathcal{L}_M, \mathcal{W})$ $(p>0)$ be the space of all continuous alternating multilinear mappings from $\mathcal{L}_M \times \cdots \times \mathcal{L}_M$ ($p$ times) to $\mathcal{W}$ and $C^0(\mathcal{L}_M, \mathcal{W})=\mathcal{W}$. The differential