36. *A Complex Analogue of the Generalized Minkowski Problem*

By Mikio ISE*)

(Communicated by Kunihiko KODAIRA, M. J. A., Sept. 12, 1977)

1. Recently, A. V. Pogorelov [5, 6] announced to have solved the generalized Minkowski problem using the idea of E. Calabi, as was also mentioned in our lecture [3]. It was a key point of solving this problem to reduce it to finding solutions of certain non-linear elliptic partial differential equations defined over the unit sphers S^n $(n\geq 2)$, which we called in [3] as *of the generalized Monge-Ampère type*. In the present note we will show that the framework of finding solutions of the differential equation mentioned above can be applied analogously also in the case of n-complex projective space P^n_C $(n\geq 1)$, instead of the unit sphere. To describe our motivation of studies, we have first to resume and explain the differential equations over S^n appearing in the generalized Minkowski problem which suits to our purpose.

Namely, we denote by ϕ the unknown C^∞-function of n-variables $u_1, u_2, \cdots, u_n$, that is in reality defined over the whole S^n; in fact, if we write the current co-ordinates of the ambient euclidean space $\boldsymbol{R}^{n+1}$ as $(\xi_0, \xi_1, \cdots, \xi_n)$ and cover S^n by the co-ordinates patches $U_i=\{\xi_i\neq 0\}$ $(0\leq i\leq n)$. In every U_i, we put $u_1=\xi_0/\xi_i$, $u_2=\xi_1/\xi_i$, $\cdots$, $u_n=\xi_n/\xi_i$, whereby one considers the differential operator D_i:

$$D_i(\phi)=|\xi_i|^{-n-2}\det\left(\frac{\partial^2\phi}{\partial u_j\partial u_k}\right)\qquad(0\leq i\leq n),\tag{1}$$

then D_i $(0\leq i\leq n)$ yield the differential operator D defined globally over the sphere S^n. The generalized Minkowski problem for an n-dimensional compact, convex oriented hypersurface V $(n\geq 2)$ is concerned with the following partial differential equation on S^n:

$$D(\phi)=\kappa,\tag{2}$$

where a given positive function κ on S^n is assumed to satisfy the conditions:

$$\int_{S^n}\kappa\cdot\xi_i dS=0\qquad(0\leq i\leq n),$$

dS denoting the volume element of S^n with respect to the natural metric of S^n (The equation (2) has been known from old times, when $n=2$, as the simplest form of the so-called Monge-Ampère equations [3]). In the

*) Prof. M. Ise died on March 16, 1977, at the age of 44, by a railway accident.