35. *Congruences of the Eigenvalues of Hecke Operators*

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Introduction. This note is a continuation of our previous note on the divisibility by 2 of the eigenvalues of Hecke operators [1]. We will omit the proofs of the theorems in this note. Details will appear in K. Hatada "On the eigenvalues of Hecke operators" [3].

§ 1. Let S_{w+2} be the space of cusp forms of weight $w+2$ on $SL(2,Z)$. Let λ_p be any eigenvalue of the Hecke operator $T(p)$ on S_{w+2} where p is a rational prime. In K. Hatada [1] we proved the following Theorem 1 and announced Theorem 2:

Theorem 1. *λ_p is divisible by 2 for any rational prime p and for any even weight $w+2$.*

Theorem 2. (i) *λ_p is divisible by 4 for any prime p with $p\equiv -1$* mod 4 *and for any even weight $w+2$.*

(ii) *(λ_p-2) is divisible by 4 for any prime p with $p\equiv +1$* mod 4 *and for any even weight $w+2$.*

Prof. J.-P. Serre sent us some experimental results, computed on a machine, which are proved by Theorems 2, 4 and 5 in this note.

Later he sent his conjectures compatible with the known results (see Remark 1 below), which are proved by Theorems 3 and 6. The author wishes to express his gratitude to Prof. Serre for his suggestions.

In § 1 of this note we give congruences for eigenvalues of the Hecke operators on S_{w+2}. They are Theorems 3–9.

Let λ_p be any eigenvalue of the $T(p)$ on S_{w+2}.

Theorem 3. $\lambda_p\equiv 1+p \bmod 8$, *for any odd prime p and for any even weight $w+2$.*

Theorem 4. *λ_2 is divisible by 8 for any even weight $w+2$.*

Theorem 5. (i) *λ_2 is divisible by 16 for any weignt $w+2$ such that $w\equiv 0$* mod 4.

(ii) *λ_2 is divisible by 32 for any weight $w+2$ such that $w\equiv 0$* mod 4 *and $w\not\equiv 0$* mod 8.

Theorem 6. $\lambda_p\equiv 1+p \bmod 3$ *for any rational prime p except for $p=3$ and any even weight $w+2$.*

Theorem 7. *λ_3 is divisible by 3 for any even weight $w+2$.*

Theorem 8. $\lambda_{11}\equiv 2 \bmod 5$ *for any even weight $w+2$.*

Theorem 9. $\lambda_{19}\equiv 0 \bmod 5$ *for any even weight $w+2$.*

Remark 1. Let tr $T(p)_{w+2}$ be the trace of the $T(p)$ on S_{w+2}. A few