29. *Fundamental Solutions to the Cauchy Problem of Some Weakly Hyperbolic Equation*

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1. Consider the operator

$$L=D_t^2-t^{2m}\sum_{j,k=1}^n a_{jk}D_jD_k+b_0D_t+\sum_{j=1}^n b_jD_j+c.$$

Here m is a positive integer, and $a_{jk}=a_{jk}(t,x)$, $b_l=b_l(t,x)$, $c=c(t,x)C^\infty$ functions of $(t,x)=(t,x_1,\cdots,x_n)\in \boldsymbol{R}\times\boldsymbol{R}^n$. $D_t=-i\partial/\partial t$, $D_j=-i\partial/\partial x_j$, $j=1,\cdots,n$, and $i^2=-1$ as usual. We assume that $(a_{jk}(t,x))$ be a real symmetric positive definite matrix, reducing to the unit matrix for t,x sufficiently large.

2. Let $\tau\in\boldsymbol{R}$. Consider the following Cauchy problem:

$$(*)\qquad \begin{cases} Lv(t,x)=0, & t>\tau,\ x\in\boldsymbol{R}^n,\\ v(t,x)|_{t=\tau}=f_0(x), & D_tv(t,x)|_{t=\tau}=f_1(x),\end{cases}$$

f_0, f_1 being given distributions in $\mathcal{E}'(\boldsymbol{R}^n)$.

Let $\Delta=\{(t,\tau);\tau\leqq t\}$.

Definition. Let $U_j(t,\tau)$, $j=0,1$, be operators from $\mathcal{E}'(\boldsymbol{R}^n)$ to $\mathcal{D}'(\boldsymbol{R}^n)$ with kernels in $C^\infty(\Delta;\mathcal{D}'(\boldsymbol{R}^n\times\boldsymbol{R}^n))$. We call $U_j(t,\tau)$, $j=0,1$, a pair of fundamental solutions to the problem $(*)$ if

$$LU_j(t,\tau)=0,\ j=0,1,\qquad \text{in } \Delta,$$
$$D_t^kU_j(t,\tau)|_{t=\tau}=\delta_{jk}I,\qquad j,k=0,1,$$

δ_{jk} being the Kronecker symbol and I the identity operator.

3. The purpose of the present note is to construct a pair of fundamental solutions to the problem $(*)$ under the conditions explained below. We set

$$a(t,x,\xi)=\Big(\sum_{j,k=1}^n a_{jk}(t,x)\xi_j\xi_k\Big)^{1/2},\qquad \xi\in\boldsymbol{R}^n\backslash 0,$$

so that the principal symbol of L is

$$L_0(t,x,\xi_0,\xi)=(\xi_0-t^m a(t,x,\xi))(\xi_0+t^m a(t,x,\xi)).$$

We denote by $S_L(t,x,\xi_0,\xi)$ the subprincipal symbol of L. Thus,

$$S_L(t,x,\xi_0,\xi)=b_0(t,x)\xi_0+\sum_{j=1}^n b_j(t,x)\xi_j + it^{2m}\sum_{j,k=1}^n \xi_k\partial a_{jk}(t,x)/\partial x_j.$$

4. Set

$$C_{L\pm}(t,x,\xi)=S_L(t,x,\pm t^m a(t,x,\xi),\xi).$$

We assume

$$(1)\qquad C_{L\pm}(t,x,\xi)=t^{m-1}b(x,\xi)+t^m b_\pm(t,x,\xi).$$

Here $b(x,\xi)$ and $b_\pm(t,x,\xi)$ are smooth functions of t,x,ξ. For simplicity, we require that $\operatorname{Im}\{b(x,\xi)/|\xi|\}$ be uniformly bounded on $\boldsymbol{R}^n\times(\boldsymbol{R}^n\backslash 0)$.

5. Theorem. *Under the assumption* (1), *there exists a unique*