86. *Probability-theoretic Investigations on Inheritance. II_2. Cross-Breeding Phenomena.*

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3. Inhomogeneous distribution.

A population with a mosaic composition, discussed in two preceding sections, may be regarded as a sort of population with *inhomogeneous*, i.e. *non-uniform* distribution. Let us now consider a general type of inhomogeneous distributions. Now, generally in inheritance phenomena, the unit of observations is an individual. Hence, if we attempt to introduce the concept of distribution-density of various quantities, it will be reprehensible to divide the region immoderately to pieces. But, as in case of usual population-density, we may define the concept of *distribution-density* suitably according to such a moderate division that the probability-theoretic considerations are expected to apply appropriately.

We consider a region G in which the inheritance phenomenon with genes $A_i (i = 1, \dots, m)$ will be observed. Suppose that G is a region on a plane and the position of each point of G is denoted by coordinates (x, y) with respect to a suitably chosen rectanular coordinate system.——In case where G is a region on a surface, the subsequent discussion will also remain valid with slight modifications; in fact, we have only to choose a suitable curvilinear coordinate system and correspondingly to make use of its surface element instead of $dxdy$.

Let the distribution-density of the phenotypes A_{ij} in the region G be denoted (at least approximately) by a set of contiuous or, more generally, piecewise continuous functions

$$(3.1) \qquad P_{ij}(x, y) \quad (i, j = 1, \dots, m; i \leqq j) \\ (P_{ij}(x, y) = P_{ji}(x, y)).$$

Let the distribution-density of the genes A_i be denoted by

$$(3.2) \qquad p_i(x, y) \quad (i = 1, \dots, m).$$

Then, at each point of G, the fundamental relations

$$(3.3) \qquad \sum_{i=1}^{m} p_i(x, y) = 1, \quad \sum_{i \leqq j} P_{ij}(x, y) = 1$$

must hold. In any small part the distribution being regarded to be in an equilibrium state, we have the interrelations

$$(3.4) \quad P_{ii}(x, y) = p_i(x, y)^2, \quad P_{ij}(x, y) = 2p_i(x, y)p_j(x, y) \quad (i \neq j),$$