60. On the fundamental differential equations of flat projective geometry.

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§ 1. In an adress given at the International Mathematical Congress at Bologna,[1] Prof. O. Veblen defined the geometry as the theory of an invariant. According to the Erlanger Programm a geometry is the invariant theory of a group. According to the new conception of O. Veblen a geometry is the theory of invariant.

O. Veblen showed, in a lecture to the London Mathematical Society,[2] that the classical projective geometry may be regarded as the theory of an invariant subject to certain restrictions. The theory of this invariant free from these restrictions is the so-called generalized projective geometry.

If we introduce a curvilinear coordinate system (ξ^i)[3] in an n-dimensional projective space, the homogeneous projective coodinates Z^λ of the space may be expressed in the form

$$Z^\lambda = e^{\xi^0} p^\lambda_\mu f^\mu(\xi^i), \tag{1.1}$$

where e^{ξ^0} is an arbitrary factor, p^λ_μ constants subject only to the condition that the determinant $|p^\lambda_\mu|$ formed with p^λ_μ is defferent from zero and finally $f^\lambda(\xi)$ $n+1$ analytic functions of ξ^i such that the determinant

$$\begin{vmatrix} f^0, & f^1, & \dots, & f^n \\ \frac{\partial f^0}{\partial \xi^1}, & \frac{\partial f}{\partial \xi^1}, & \dots, & \frac{\partial f^n}{\partial \xi^1} \\ \dots & \dots & \dots & \dots \\ \frac{\partial f^0}{\partial \xi^n}, & \frac{\partial f^1}{\partial \xi^n}, & \dots, & \frac{\partial f^n}{\partial \xi^n} \end{vmatrix} \neq 0. \tag{1.2}$$

Differentiating (1.1) and eliminating the constants, we find that any $n+1$ homogeneous projective coordinates Z^λ defined as functions of curvilinear coordinates ξ^i must satisfy the differential equations

(1) O. Veblen: Differential invariants and geometry. Atti Congresso Internazionale Bologna, **1**, (1928), 181–189.

(2) O. Veblen: Generalized projective geometry. Journal of the London Math. Soc., **4** (1929), 140–160.

(3) Greek indices take the values on the range 0, 1, 2, ..., n and Roman indices on the range 1, 2, ..., n.