26. *Fundamental Theory of Toothed Gearing (IV).*

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(Comm. by T. KUBOTA, M. J. A., May 12, 1949.)

We have developed the general theory of profile curves in the preceding reports from (I) to (III).[1] Now we shall give its several applications to practical curves.

§ 1. Profile curves of cycloidal system.

Take a circle with radius a_γ as a rolling curve K_γ. However, in this case, as a pitch curve K we may not necessarily take a circle. Suppose that K_γ (and accordingly K) is oriented as a_γ is positive, that is, the direction of K_γ is positive, if the center O of the circle K_γ always exists on the left side to the direction. From the two points at which the straight line connecting the center O_γ of K_γ with a drawing point C invariably connected with K_γ intersects the perimeter of K_γ we choose the nearer one to C, denoting it by P_0 and adopt P_0 as origin. And denote by s the length of arc measured from the origin to an arbitrary point P on K_γ. Denote by r the signed length of the segment PC and by θ the angle between the straight line PC and the tangent to K_γ at P, where $sgn(\theta) = sgn(r)$.

If we find the relation $r=f(s)$ between r and s and the relation $r=g(\theta)$ between r and θ, they are respectively the equations of the profile curve F drawn by the drawing point C and of the path of contact Γ corresponding to F.

Now from the triangle $O_\gamma PC$ we have

$$PC^2 = O_\gamma C^2 + O_\gamma P^2 - 2O_\gamma C \cdot O_\gamma P \cos C\hat{O}_\gamma P$$

and then denoting by e the length of the spgment P_0C

$$r^2 = e^2 + 4a_\gamma(a_\gamma - e)\sin^2 \frac{s}{2a_\gamma}$$

Hence, when $e>0$

$$(1)_1 \qquad r = f(s) = \sqrt{e^2 + 4a_\gamma(a_\gamma - e)\sin^2 \frac{s}{2a_\gamma}}$$

and when $e<0$

$$(1)_2 \qquad r = f(s) = \begin{cases} -\sqrt{e^2+4a_\gamma(a_\gamma-e)\sin^2\frac{s}{2a_\gamma}}, & \text{where } |s| \leqq a_\gamma \cos^{-1}\left(\frac{a_\gamma}{a_\gamma - e}\right) \\ \sqrt{e^2+4a_\gamma(a_\gamma-e)\sin^2\frac{s}{2a_\gamma}}, & \text{where } |s| \geqq a_\gamma \cos^{-1}\left(\frac{a_\gamma}{a_\gamma - e}\right) \end{cases}$$

In particular, when $e=0$, that is, the drawing point C exists on the perimeter of K_γ,

1) This Proceedings, Vol. 25 (1949). No. 2.