22. On the Behaviour of the Boundary of Riemann Surfaces. I.

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We shall define the generalized harmonic measure of the boundary of a given Riemann surface and then classify it into two types.

Theorem 1. (*An extension of R. Nevanlinna's theorem*[1]) *Let F be a Riemann surface with a finite number of sheets spread over the z-plane and have Green's function and E be the set of its all accesible boundary points. We map the universal covering surface F^∞ of F on $|w|<1$, then the measure of e on $|w|=1$, which corresponds to E, is 2π.*

Proof. The mapping function $z=f(w)$ is automorphic with respect to a Fuchsian group G and let F be mapped on a fundamental domain D_0, which contains $w_0=0$. Let w_n be its equivalent. Since F has Green's function, we have by Poincaré's theorem[2]

$$\sum_{n=0}^{\infty}(1-|w_n|)<\infty\,. \tag{1}$$

We shall show that characteristic function $T(r)$ of $f(w)$ is bounded. Let a_0 be any point in D_0 and a_n be its equivalent, then we have

$$\left|\frac{w_0-a_0}{1-\bar{a}_0\cdot w_0}\right|=\left|\frac{w_n-a_n}{1-\bar{a}_n\cdot w_n}\right|.$$

Hence

$$|a_0|=\left|\frac{0-a_0}{1-\bar{a}_0\cdot 0}\right|=\left|\frac{w_n-a_n}{1-\bar{a}_n\cdot w_n}\right|.$$

From this we can easily deduce that

$$\frac{1-|a_0|}{1+|a_0|}(1-|w_n|)\leqq 1-|a_n|\leqq\frac{1+|a_0|}{1-|a_0|}(1-|w_n|)\,. \tag{2}$$

With respect to a general meromorphic function $f(w)$, $N(r,a)$ and $\sum_{r_\nu\leqq r}(1-r_\nu(a))$, where $r_\nu(a)$ is the absolute value of a-point of $f(w)$, are reciprocally uniformly bounded for some set of a.

Now we apply it on our automorphic function $f(w)$. Denote by $n(z)$ the number of sheets of F above z. Since F consits of a finite number of sheets, the maximum of $n(z)$ when z varies on the

1) R. Nevanlinna: Eindeutige Analytische Funktionen. Berlin, (1936), p. 204.

2) H. Poincaré: Sur l'uniformisation des fonctions analytiques. Acta. Math, 31 (1907).