30. Notes on Fourier Analysis (XL): Remark on the Rademacher System.

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§ 1. Let $\{r_n(x)\}$ denote the Rademacher system, and let $\{p_n\}$ $(n = 1, 2, \ldots)$ be an increasing sequence of positive numbers. If we denote $P_n = p_1 + p_2 + \cdots + p_n$ and

$$(1) \qquad \varphi_n(x) = [p_1 r_1(x) + p_2 r_2(x) + \cdots + p_n r_n(x)]/P_n$$

$(n = 1, 2, \ldots)$, the following theorems are known (J. D. Hill [1]):

(i) *The set of convergence points of* $\varphi_n(x)$ *is of measure* 0 *or* 1[1]; (ii) *if the series*

$$(2) \qquad \sum_{n=1}^{\infty} (p_n/P_n)^2$$

converges, then $\varphi_n(x)$ *converges to zero almost everywhere; and conversely* (iii) *if* $\varphi_n(x)$ *converges in a set of positive measure, its limit is necessarily zero almost everywhere, and moreover*

$$(3) \qquad \lim_{n\to\infty} p_n/P_n = 0.$$[2]

Let us consider now the condition which implies the convergence almost everywhere of $\varphi_n(x)$. It is also known that the condition (3) is insufficient to assert such convergence (G. Maruyama [4] and the author [6]). In this note, by determining the decreasing order of (3), we shall give a sufficient condition different from the convergence of (2)[3].

THEOREM. *If we have*

$$(4) \qquad p_n/P_n = o(1/\log\log P_n) \qquad \text{as } n \to \infty,$$

then $\varphi_n(x)$ *converges to zero almost everywhere.*

The condition (4) is the best possible one of this form, in fact, there exists an increasing sequence of positive numbers $\{p_n\}$ such that $p_n/P_n = O(1/\log\log P_n)$ as $n \to \infty$, and $\varphi_n(x)$ diverges almost everywhere. An example with this property was furnished by Mr.

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1) We shall understand, throughout this paper, that the sets are included in (0, 1), that is, $0 < x < 1$.

2) Cf. Remark 3, § 3.

3) Cf. Remark 4, § 3.