28. On the Simple Extension of a Space with Respect to a Uniformity. II.

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(Comm. by K. KUNUGI, M.J.A., March 12, 1951.)

The present note is a continuation of our previous study concerning the simple extension of a space with respect to a uniformity[1]. As an application we deduce Shanin's theory on the bicompact extensions of topological spaces[2]. We use the same terminologies and notations as in the first note which will be cited with I.

§ 1. **A characterization of the simple extension.** Let R^* be the simple extension of a *space* R with respect to a *uniformity* $\{\mathfrak{U}_\alpha;\ \alpha \in \Omega\}$[3]. Then we have

Lemma 1. *For an open set G of R it holds that $G^* = R^* - \overline{R-G}$, where the bar indicates the closure operation in R^*.*

Proof. Since $(R-G)\cdot G^* = 0$ by I, Lemma 5, we have $R-G \subset R^*-G^*$ and hence $\overline{R-G} \subset R^*-G^*$. On the other hand, if $x \in R^*-G^*$, then, for any open set H of R such that $x \in H^*$, we have $H^*(R^*-G^*) \neq 0$, and hence $H^*(R-G) \neq 0$; this shows that $R^*-G^* \subset \overline{R-G}$.

Theorem 1. *The simple extension R^* of a space R with respect to a uniformity $\{\mathfrak{U}_\alpha;\ \alpha \in \Omega\}$ is characterized as a space S with the following properties (i.e. such a space S is mapped on R^* by a homeomorphism which leaves each point of R invariant):*

(1) *R is a subspace of S.*

(2) *$\{S-\overline{R-G};\ G$ open in $R\}$ is a basis of open sets of S.*

(3) *Each point of $S-R$ is closed.*

(4) *$\mathfrak{V}_\alpha = \{S-\overline{R-U};\ U \in \mathfrak{U}_\alpha\}$ is an open covering of S.*

(5) *$\{S(x, \mathfrak{V}_\alpha);\ \alpha \in \Omega\}$ is a basis of neighbourhoods at the point x of $S-R$.*

(6) *For any point x of $S-R$ there exists a vanishing Cauchy family $\{X_\lambda\}$ of R (with respect to $\{\mathfrak{U}_\alpha\}$) such that $x = \Pi\overline{X}_\lambda$ in S, and*

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1) K. Morita: On the simple extension of a space with respect to a uniformity, I, the Proc. **27**, No. 2 (1951).

2) N. A. Shanin: Doklady URSS, **38** (1943), pp. 3-6; pp. 110-113; pp. 154-156. These papers are not yet accessible to us; we knew the results only by Math. Reviews.

3) Cf. I, §§ 1 and 3. It is to be noted that a space means here a neighbourhood space such that the family of all open sets containing a point p forms a basis of neighbourhoods of p.