27. Theorems on the Convexity of Bounded Functions.

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(Comm. by K. KUNUGI, M.J.A., March 12, 1951.)

§ 1. Introduction.

We denote by R_M the family of functions $\{F_{(z)}\}$ which are regular in $|z|<1$ and have the properties

$$|F(z)|\leqq M \ (M\geqq 1),\ F(0)=0,\ F'(0)=1,$$

and by S_M the family of functions $\{F(z)\}$ which belong to R_M and schlicht in $|z|<1$.

Dieudonne[1] has proved that any function $F(z)$ of the class R_M is schlicht in $|z|<M-\sqrt{M^2-1}$ and this circle is transformed into a starshaped region in w-plane by $w=F(z)$ and the number $M-\sqrt{M^2-1}$ cannot be replaced by any greater one, and R. Nevanlinna[2] has proved that, for any function $F(z)$ which is regular, schlicht in $|z|<1$ and has the properties $F(o)=0$, $F'(o)=1$, the "Rundungsschranke" is $2-\sqrt{3}$.

In this paper, we will find the greatest circle in which any function $F(z)$ of the class R_M is convex, and the "Rundungsschranke" of the class S_M. For this purpose we will show some lemmas in § 2 and will treat the problems cited above in § 3 and 4.

§ 2. Lemmas.

Let $F(z)$ be any function of the class R_M, then

Lemma 1

$$M|z|\frac{1-M|z|}{M-|z|}\leqq|F(z)|\leqq M|z|\frac{1+M|z|}{M+|z|},\qquad |z|<1.$$

Lemma 2 (Simonart)[3]

$$\frac{(M+|F(z)|)(|F(z)|-M|z|^2)}{M|z|(1-|z|^2)}\leqq|F'(z)|\leqq\frac{(M-|F(z)|)(|F(z)|+M|z|^2)}{M|z|(1-|z|^2)},\ |z|<1.$$

Lemma 3[4]

Let $F(z)=\sum_{\nu=1}^{\infty}c_\nu z^\nu$ be regular and $|F(z)|<M$ in $|z|<1$, then

$$M-\frac{|c_1|^2}{M}\geqq|c_2|.$$

For the function $F(z)$ which belongs to the class S_M, the function of ζ.