26. *Generalisation of R. Baire's Theorem on Differential Equation* $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}f(x,y)=0.$

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Introduction.

In this paper, we shall denote by G a fixed open set in R^2 (Euclid plane defined by two coordinates x, y), by $f(x, y)$ a fixed continuous function defined everywhere in G, which has continuous f_y. (Functions will be always real-valued in this paper.)

We shall consider the partial differential equation

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}f(x, y)=0. \tag{1}$$

With (1), we shall associate the ordinary differential equation

$$\frac{dy}{dx}=f(x, y). \tag{2}$$

The curves representing the solutions of (2) which are prolonged up to the boundary of G on both sides and cannot be prolonged further, will be called characteristic curves (characteristics).

Through any point (x_0, y_0) in G, there passes one and only one characteristic curve $y=y(x, x_0, y_0)$.

(For the precise meanings of the propositions and their proofs cf. 1)).

We know[2)] that a continuous function $z(x, y)$ defined in G, is constant on each characteristic curve, if $z(x, y)$ has continuous $\partial z/\partial x$, $\partial z/\partial y$ or more generally is totally differentiable in G and satisfies (1) in G.

R. Baire showed in his thesis[3)] that, in the above, the assumption of continuity of $\partial z/\partial x$, $\partial z/\partial y$ or total differentiability of $z(x, y)$ is superfluous and, instead of them, only the existence of $\partial z/\partial x$, $\partial z/\partial y$ is sufficient.

We shall call it Baire's theorem.

In this paper we shall give a new proof of Baire's theorem by a method entirely different from his and somewhat generalise it.