50. *On the Hauptsehne of the Region to which the Unit-Circle is Mapped by the Bounded Function.*

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(Comm. by K. KUNUGI, M.J.A., May 16, 1951.)

Let $F(z)(F(0)=0,\ F'(0)=1)$ be regular and schlicht in $|z|<1$ and B the mapped region of $|z|<1$ by $w=F(z)$. Then G. Szegö has proved that, for the length l of any Hauptsehne of the region B, the inequality,

$$l \geqq 1 \tag{1}$$

holds true.

In this note we will show that the relation (1) can be replaced by the more exact one, if we add the property that $F(z)$ is the bounded function, i. e.

$$|F(z)|<M \quad (M \geqq 1).$$

If r_1, r_2 are the boundary points on the Hauptsehne of the region B and lie on the opposite side with respect to the origin $w=0$, then

$$\arg r_2 = \arg r_1 + \pi.$$

Therefore,

$$l = |r_2 - r_1| = |r_1| + |r_2|,$$

where l denotes the distance of two points r_1, r_2.

The function

$$\phi(z) = \frac{r_1 M^2 F(z)}{[r_1 - F(z)][M^2 - \varepsilon r_1 F(z)]}, \quad |\varepsilon| = 1,$$

is normalized and regular, schlicht in $|z|<1$ and

$$\frac{M^2 r_1 r_2}{[r_1 - r_2][M^2 - \varepsilon r_1 r_2]}$$

is the boundary point of the mapped region of $|z|<1$ by $w=\phi(z)$. Hence, by the theorem due to Koebe, we have

$$\left| \frac{M^2 r_1 r_2}{(r_1 - r_2)(M^2 - \varepsilon r_1 r_2)} \right| \geqq \frac{1}{4}. \tag{2}$$

Being ε arbitrary, the inequality (2) is reduced to