

49. A Theorem of Liouville's Type for Meson Equation.

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In connection with the stochastic integrability of Fokker-Planck's equation¹⁾, the author encountered with the following

Problem. Let R be a connected domain with smooth boundaries ∂R of an n -dimensional euclid space $R_n (n \geq 2)$. Does there exist, for $m > 0$, a bounded solution $h(x)$ other than 0 of the meson equation

$$(1) \quad \Delta h(x) = m h(x) \text{ in } R$$

with the boundary condition

$$(2) \quad \frac{\partial h}{\partial n} = 0 \text{ on } \partial R, \text{ } n \text{ denoting outer normal?}$$

The purpose of the present note is to give an answer to this problem of Liouville's type. It reads as follows:

Let the boundaries ∂R lie entirely in the bounded part of R_n , and let $m(x)$ be a continuous function such that

$$(3) \quad \inf_{x \in R} m(x) = m > 0.$$

Then the solution $h(x)$ of

$$(1) \quad \Delta h(x) = m(x) h(x) \text{ in } R$$

satisfying the boundary condition (2) together with the order relation

$$(4) \quad h(x) = O(\exp(\alpha|x|)), \text{ where } 2\alpha < \sqrt{2m},$$

must vanish identically. Here $|x|$ denotes the distance of the point x from the origin of R .

Proof. 1st case (R is a bounded domain). By (2) and the Green's integral theorem, we have

$$m \int_R h(x)^2 dx \leq \int_R h(x) \Delta h(x) dx \leq \int_{\partial R} h(x) \frac{\partial h}{\partial n} dS = 0.$$

Here dx and dS respectively denote the volume element of R and the hypersurface element of ∂R . Thus $h(x)$ must $\equiv 0$.

1) K. Yosida: Integration of Fokker-Planck's equation with a boundary condition, Journal of the Mathematical Society of Japan, Takagi's Congratulation Number. The result obtained below implies the existence of the Brownian motion in R with ∂R as reflecting barrier.