47. *A Condition for an Abelian Group to be a Free Abelian Group with a Finite Basis.*

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1. Let G be a countable abelian (additive) group. An integer-valued function $f(\xi, \eta)$ on $G\times G$ is said to be bilinear if

$$f(\xi_1+\xi_2, \eta_1+\eta_2) = \sum_{i,j} f(\xi_i, \eta_j)$$

for any elements $\xi_i, \eta_j, (i, j = 1, 2)$.

Put

$$G_p = \{p\xi ; \xi \in G\},$$

where p is a fixed integer.

Prof. Igusa conjectured that *an abelian group G is a free abelian group with a finite basis if a bilinear integer-valued function $f(\xi, \eta)$ is defined on $G\times G$ which vanishes only at the identity element of $G\times G$, and if G/G_p is a finite group.*

The purpose of this note is to give an affirmative answer. We shall prove the

Theorem. *An abelian group G satisfying the above conditions is a free abelian group with a finite basis.*

2. From the above condition concerning the bilinear function $f(\xi, \eta)$ we can easily deduce that

(i) *there does not exist an element of finite order,*

(ii) *there does exist only a finite number of elements which are the divisors of a fixed element.*

An element is said to be prime if the element has not any divisor except itself. The set of prime elements $\{\xi_i\}$ is called a canonical system if for relative prime integers a_i the element

$$a_1\xi_1+a_2\xi_2+\cdots+a_s\xi_s$$

is also prime. A subgroup which is spanned by a canonical system is clearly a free abelian group. We shall now prove the following lemma.

Lemma. Let Ξ be a subgroup of rank $s-1$, spanned by a canonical system $\{\xi_1, \xi_2, \ldots, \xi_{s-1}\}$. If $G\neq\Xi$ there exists a canonical system $\{\varphi_1, \varphi_2, \ldots, \varphi_s\}$ such that the subgroup Φ spanned by the $\{\varphi_i\}$ contains Ξ and the rank of Φ is equal to s.