61. On the Equi-Continuity in Semi-Ordered Linear Spaces.

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Let a semi-ordered linear space R be universally continuous and semi-regular. The *equi-continuity* and *universally equi-continuity* are defined for a system of elements of R by H. Nakano in a book[1], and it was proved in it that these concepts become equivalent for a system of countable elements. We will prove in this paper that this equivalency holds for every system. In the sequel we shall employ notations in the book cited above.

Let $\bar{R}$ be the conjugate space of R. For a manifold K of R we define a functional μ_K on $\bar{R}$ such that $\mu_K(\bar{a}) = \sup_{x \in K} |\bar{a}|(|x|)$ for every $\bar{a} \in \bar{R}$, then we can see easily that we have for every $\bar{a}$, $\bar{b} \in \bar{R}$

1) $0 \leqq \mu_K(\bar{a}) \leqq +\infty$,
2) $|\bar{a}| \leqq |\bar{b}|$ implies $\mu_K(\bar{a}) \leqq \mu_K(\bar{b})$,
3) $\mu_K(\alpha\bar{a}) = |\alpha| \mu_K(\bar{a})$ for every real number α,
4) $\mu_K(\bar{a}+\bar{b}) \leqq \mu_K(\bar{a}) + \mu_K(\bar{b})$.

If we denote by $\bar{M}_K$ the set of all elements $\bar{a}$ of $\bar{R}$ such that $\mu_K(\bar{a}) < +\infty$, then $\bar{M}_K$ is a semi-normal manifold of $\bar{R}$ and the functional μ_K is a norm on $\bar{M}_K[K]$ because if $\bar{a} \in \bar{M}_K[K]$ and $\mu_K(\bar{a}) = 0$, then since $|\bar{a}|(|x|) = 0$ for every $x \in K$, we have $|\bar{a}| = |\bar{a}|[K] = 0$.

If K is weakly bounded, then $\bar{M}_K$ coincides with $\bar{R}$ and μ_K is a norm on the normal manifold $\bar{R}[K]$, and if K is moreover equi-continuous, then this norm on $\bar{R}[K]$ is obviously a continuous norm. Since a continuous semi-ordered linear space having a continuous norm is superuniversally continuous[2], we obtain the following theorem:

Theorem. *If a manifold K of R is equi-continuous, then $\bar{R}[K]$ is superuniversally continuous and for any $\bar{a}_\lambda \downarrow_{\lambda \in \Lambda} 0$, $\bar{a}_\lambda \in \bar{R}$ and real number $\varepsilon > 0$ there exists $\lambda_0 \in \Lambda$ for which we have $\bar{a}_{\lambda_0}(|x|) \leqq \varepsilon$ for every $x \in K$.*

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1) H. Nakano: Modulared Semi-ordered Linear Spaces, Tokyo Mathematical Book Series vol. I (1950) p. 109.

2) The book cited above, theorem 30. 7.