60. *Theorems on the Cluster Sets of Pseudo-Analytic Functions.*

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Let D be a domain on the z-plane and C be its boundary. Let E be a bounded closed set of capacity[1] zero, included in C and z_0 be a point in E. Let $w=f(z)$ be a single-valued function pseudo-analytic in D. The cluster set $S_{z_0}^{(D)}$ is the set of all values α such that $\alpha=\lim_{n\to\infty} f(z_n)$, where z_n $(n=1, 2, \dots)$ is a sequence of points tending to z_0 inside D. The cluster set $S_{z_0}^{*(C)}$ is the intersection of the closure of the union $US_{z'}^{(D)}$ for all z' belonging to the part of $C-E$, which lies in $|z-z_0|<r$.

Since E is of capacity zero, by Evan's theorem[2], we can distribute a positive measure $d\mu(a)$ on E such that its potential

$$u(z)=\int_E \log\frac{1}{|z-a|}d\mu(a), \qquad \int_E d\mu(a)=1$$

is harmonic outside E, excluding $z=\infty$, and has boundary value $+\infty$ at any point of E. Let $v(z)$ be its conjugate harmonic function and put

$$\zeta=\zeta(z)=e^{u(z)+iv(z)}=r(z)e^{iv(z)}=re^{i\theta}.$$

The niveau curve $C_r: r(z)=\text{const.}=r\ (0<r<+\infty)$ consists of a finite number of Jordan curves surrounding E. Let J_r be its component which surrounds z_0. Let V_r be the closure of the set of all values taken by $f(z)$ in the part of D, which lies in the interior of J_r. Then $S_{z_0}^{(D)}$ is identical with the intersection of all V_r. Let M_r be the closure of the union $US_{z'}^{(D)}$ for all z' belonging to the part of $C-E$, which lies in the interior of J_r. Then $S_{z_0}^{*(C)}$ is identical with the intersection of all M_r. *Let (P) denote the class of functions $w=f(z)$ which are single-valued and pseudo-analytic in D and for which the integral*

$$\int^{\infty}\frac{dr}{rD(r)} \tag{1}$$

diverges, where $D(r)$ is the smallest upper bound of the 'Dilatationsquotient'[3] $D_{z|w}$ of $w=f(z)$ on the part of C_r which lies in D.

1) 'Capacity' means logarithmic capacity in this paper.
2) G. C. Evans: Monatshefte f. Math. u. Phys. 43 (1936).
3) O. Teichmüller: Deutsche Math. 3 (1938).