

## 74. On Some Representation Theorems in an Operator Algebra. I.

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I. E. Segal has proved that a state of a  $C^*$ -algebra is the normalizing function of some normal representation (cf. [2]<sup>1)</sup>). ( $C^*$ -algebra is a uniformly closed self-adjoint operator algebra on a Hilbert Space, in the terminology of I. E. Segal.) Applying the Reduction Theory of J. von Neumann (cf. [1]) for this theorem, we can see that a state of a separable  $C^*$ -algebra is a directed integral of a system of pure states, and we can see a similar result for trace instead of state (the terminologies of state, pure state and trace etc. are of I. E. Segal [2] and M. Nakamura [3] respectively). This is a Theorem of Bochner's type in  $*$ -algebra. From this, we can easily see by a topological method the Bochner's Theorem for a separable locally compact group. (Recently, this theorem has been shown by F. I. Mautner [6].)

M. Nakamura [3] has introduced the two-sided representation of a  $C^*$ -algebra which is a generalized form of double unitary representation in the sense of R. Godement [8]. From his formulation, we can see that a two sided representation of a  $C^*$ -algebra is a directed integral of a system of irreducible two-sided representations. From this fact and the Bochner's Theorem, any two-sided continuous unitary representation in a separable unimodular locally compact group is a directed integral of a system of irreducible two sided continuous unitary representations, it follows the same type theorem of F. I. Mautner [6] for one-sided continuous unitary representation of the group.

We shall describe in this paper only on a weight function  $\sigma(\lambda)$  which generates the irreducible factors. But it may be possible to prove a decomposition for any  $N$ -function in the sense of von Neumann (cf. [1]) as a weight function.

Throughout this paper, we shall assume the separability axiom, because we shall use the Reduction Theory of J. von Neumann.

1. A Bochner's type Theorem in a  $C^*$ -algebra. Recently, the Theorem of this type has been proved for the case of non-separable central  $C^*$ -algebra by M. Nakamura—Y. Misonou [4], and for the

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