

72. The Order of the Derivative of a Meromorphic Function.

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The following result is due to Whittaker¹⁾:

Theorem. *Any meromorphic function is of the same order as its derivative.*

Whittaker's own proof of the theorem was based upon a result concerning the expansion of a meromorphic function into a series of Mittag-Leffler's type which had also been established by himself²⁾. He further remarked in the addenda³⁾ at the end of the Journal containing his paper that Valiron drew his attention to a memoir⁴⁾ in which Valiron had previously proved the theorem. But, in the Valiron's paper we can find no detail; in fact, only the following statement is found there:

Signalons encore proposition: *l'ordre ρ d'une fonction méromorphe $f(z)$ et l'ordre de sa dérivée sont égaux.* C'est évident lorsque f est le quotient d'une fonction entière f_1 d'ordre au plus égal à ρ par un produit canonique P d'ordre ρ et dans le cas contraire, la propriété résulte de ce que la fonction $f_1 P' - f_1' P$ est d'ordre ρ si f_1 est d'ordre ρ et P d'ordre inférieur à ρ .

Recently, Tsuji has succeeded to give a simple proof of the theorem essentially based upon Valiron's idea which will be in a paper⁵⁾ before long published. The last part of the above cited Valiron's statement will really be found in this paper as a lemma accompanied by a proof.

The purpose of the present paper is to give a more brief proof of this interesting theorem. The last part of the Valiron's statement will also be established, as a corollary of the theorem, at the end of the present paper.

Let $f(z)$ be a meromorphic function of order ρ , and let the order of its derivative $f'(z)$ be denoted by ρ' . If $f(z)$ is an integral

1) J. M. Whittaker, The order of the derivative of a meromorphic function. Journ. London Math. Soc. **11** (1936), 82-87.

2) J. M. Whittaker, A theorem on meromorphic function. Proc. London Math. Soc. (2) **40** (1935), 255-272.

3) J. M. Whittaker, Addendum to the previous paper. Journ. London Math. Soc. **11** (1936), 320.

4) G. Valiron, Sur la distribution des valeurs des fonctions méromorphes. Acta Math. **47** (1926), 117-142.

5) M. Tsuji, On the order of the derivative of a meromorphic function. Tôhoku Math. Journ. (2) **3** (1951).