151. Probability-theoretic Investigations on Inheritance. V$_2$. Brethren Combinations.

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2$^{\text{bis}}$. Brethren combination with different fathers.

We shall now compare the probabilities σ_0's with the corresponding ones σ's. We have shown in (5.11) to (5.15) of IV that the inequality

$$d(ij;ih,ih)\equiv\pi(ij;ih,ih)-\pi_0(ij;ih,ih)\geqq 0 \tag{2.11}$$

is valid for any triple i, j, h which may coincide each other. Hence introducing a notation

$$\delta(ih,\ ih)\equiv\sigma(ih,\ ih)-\sigma_0(ih,\ ih) \tag{2.12}$$

which corresponds to (5.10) of IV, we conclude immediately, in view of (1.2), that the similar inequality

$$\delta(ih,\ ih)\geqq 0 \tag{2.13}$$

remains valid for any pair i, h being admitted also to be coincident.

In other words, if we write down each table such that the types of the first and the second children are arranged in the same order, then every probability laid on the principal diagonal of the table on the brethren combination consisting of two children having their both parents in common is, in general, greater than the corresponding one of that on brethren combination consisting of two children having their mother alone in common. This statement is very reasonable, since it represents the fact that the tendency of resemblance between the types of brethren is stronger in the former case than in the latter.

In view of the inequalities obtained in a later part of § 5 of IV, the same is true also on phenotypes.

Corresponding to the exceptional cases stated in (5.18) and (5.19) of IV, we have at present, on the contrary, the inequalities

$$\begin{aligned}\delta(ii,\ ij)=\ \delta(ij,\ ii) &= \tfrac{1}{2}p_i^2p_j(1+p_i)-\tfrac{1}{2}p_i^2p_j(1+2p_i)\\ &= -\tfrac{1}{2}p_i^3p_j\leqq 0 \qquad (i\neq j),\end{aligned} \tag{2.14}$$

$$\begin{aligned}\delta(ih,jh)=\delta(jh,ih) &= \tfrac{1}{2}p_ip_jp_h(1+2p_h)-\tfrac{1}{2}p_ip_jp_h(1+4p_h)\\ &= -p_ip_jp_h^2\leqq 0 \qquad (j,h\neq i;\ j\neq h).\end{aligned} \tag{2.15}$$

Except those laid on the principal diagonal and those which have essentially be exhausted by (2.14) and (1.15), the remaining quantities σ's and σ_0's satisfy the equality