138. *On the Simple Extension of a Space with Respect to a Uniformity. IV.*

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The purpose of the present note is to show that any regular T_1-space containing a regular T_1-space R as a dense subset can be obtained by constructing the simple extension of R with respect to some regular uniformity[1], and to discuss some other extensions related to the simple extensions.

§ 1. Regular uniformity.

Theorem 1. *Let $\{\mathfrak{U}_\alpha; \alpha \in \Omega\}$ be a regular uniformity of a space R agreeing with the topology. Then the simple extension R^* of R with respect to $\{\mathfrak{U}_\alpha\}$ is characterized as a space S with the following properties:*

(1) *S contains R as a subspace.*

(2) *$\{S-\overline{R-G}\,;\ G$ open in $R\}$ is a basis of open sets for S.*

(3) *Each point of $S-R$ is closed.*

(4) *$\mathfrak{V}_\alpha = \{S-\overline{R-U}\,;\ U \in \mathfrak{U}_\alpha\}$ is an open covering of S.*

(5) *$\{S(x, \mathfrak{V}_\alpha)\,;\ \alpha \in \Omega\}$ is a basis of neighbourhoods at each point x of $S-R$.*

(6) *S is complete with respect to the uniformity $\{\mathfrak{V}_\alpha\,;\ \alpha \in \Omega\}$.*

Here the bar indicates the closure operation in S.

Proof. It is proved by I, Theorem 9 that R^* has the properties (1)–(6). Conversely, let S be a space with the properties (1)–(6). For any point x of $S-R$, $\{S(x, \mathfrak{V}_\alpha)\cdot R\,;\ \alpha \in \Omega\}$ is a Cauchy family with respect to $\{\mathfrak{U}_\alpha\}$ because of the regularity of $\{\mathfrak{U}_\alpha\}$, and hence for any $\alpha \in \Omega$ there exists $\beta, \gamma \in \Omega$ and $U_\alpha \in \mathfrak{U}_\alpha$ such that $S(S(x,\mathfrak{V}_\beta)\cdot R, \mathfrak{U}_\gamma) \subset U_\alpha$. Hence we have $\overline{S(x, \mathfrak{V}_\beta)\cdot R} \subset S(S(x, \mathfrak{V}_\beta)\cdot R, \mathfrak{V}_\gamma) \subset S-\overline{R-U_\alpha} \subset S(x, \mathfrak{V}_\alpha)$.

Since $\{\mathfrak{V}_\alpha\}$ agrees with the topology of S, $\{S(x, \mathfrak{V}_\alpha)\cdot R\,;\ \alpha \in \Omega\}$ is a vanishing Cauchy family of R with respect to $\{\mathfrak{U}_\alpha\}$ such that $x = \Pi\, \overline{S(x, \mathfrak{V}_\alpha)\cdot R}$. Therefore Theorem 1 follows immediately from II, Theorem 1.

Theorem 2. *Let R be a regular T-space, and let S be any regular T-space such that S contains R as a dense subspace and each point of S-R is closed. Then there exists a homeomorphism φ of S*

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1) K. Morita: On the simple extension of a space with respect to a uniformity. I, II, III, Proc., **27** (1951), 65-72; 130-137; 166-171. These notes shall be cited with I, II, III respectively. We make use of the same terminologies and notations as in these notes.