42. Probability-theoretic Investigations on Inheritance. VIII$_3$. Further Discussions on Non-Paternity Problems.

By Yûsaku KOMATU.
Department of Mathematics, Tokyo Institute of Technology and Department of Legal Medicine, Tokyo University.

(Comm. by T. FURUHATA, M.J.A., March 12, 1952.)

3. Sub-probability with respect to a type of child.

We now turn to the third standpoint stated at § 1, namely, the decomposition of whole probability of proving non-paternity into sub-probabilities with respect to a type of child.

Necessary components for the purpose have already been established. In fact, the table for $V(ij; hk)$ listed in § 2 of VII remains here also utile. The results which will anew be obtained in the present section are those derived by summing up the quantities $P(ij; hk)$ with respect to all the possible types A_{ij} of wife (mother of child), while, in (2.3) of VII, the summation has been extended over the types A_{hk} of child. We thus introduce here the quantity

$$R(ij) = \sum_{h,k} P(hk; ij), \tag{3.1}$$

the letters i, j, h, k being interchanged only for the sake of convenience.

First, in case of a homozygotic child A_{ii}, we obtain

$$\begin{aligned} R(ii) &= P(ii; ii) + \sum_{j \neq i} P(ij; ii) \\ &= p_i^3(1-p_i)^2 + \sum_{j \neq i} p_i^2 p_j (1-p_i)^2 \\ &= p_i^2(1-p_i)^2. \end{aligned} \tag{3.2}$$

Next, in case of a heterozygotic child $A_{ij}(i \neq j)$, we obtain

$$\begin{aligned} R(ij) &= P(ii; ij) + P(jj; ij) + P(ij; ij) + \sum_{h \neq i, j} (P(ih; ij) + P(jh; ij)) \\ &= p_i^2 p_j (1-p_j)^2 + p_i p_j^2 (1-p_i)^2 + p_i p_j (p_i + p_j)(1-p_i-p_j)^2 \\ &\qquad + \sum_{h \neq i, j} (p_i p_j p_h (1-p_j)^2 + p_i p_j p_h (1-p_i)^2) \\ &= p_i p_j (2 - 2(p_i+p_j) + p_i^2 + p_j^2 - 4p_i p_j + 3p_i p_j (p_i + p_j)). \end{aligned} \tag{3.3}$$

The partial sums corresponding to (3.1) to (3.3), (3.5) and (3.7) of VII now become

$$\sum_{i=1}^{m} P(ii; ii) = S_3 - 2S_4 + S_5, \tag{3.4}$$