47. *Principle of the Minimum Entropy in Information Theory*

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(Comm. by M. MASIMA, M.J.A., May 13, 1953)

1. Introduction

In the previous paper, we have found that the proper representation of the entropy of the continuous information was difficult and that the uncertainty relation had to be taken into account in order to complete this theory. However, the characteristics of the ensemble are given by its autocorrelation function or its spectral density, so that it is desirable to use them to represent its entropy. The key to solve this problem seems to be given in Shannon's paper where "the entropy loss in linear filter" is discussed[1]. This calculations are based on the theory of the filter which is relatively simpler than the theory of Wiener's R.M.S. criterion.

Although Wiener's theory is brilliant and strictly constructed, it is not in vain to rewrite it from the information theory. Because the prediction or filtering is to reduce the uncertainty of the system and hence, there is some hope to translate the idea of the R.M.S. criterion into the information theoretical representation.

2. Entropy of the Ensemble

Shannon has derived the formula representing the entropy loss which occurred when the ensemble passed through a filter with characteristic $k(\omega)$. It is written as

$$H_0 = H_I + \frac{1}{2\pi W}\int_W \log |k(\omega)|^2 \, d\omega \,, \tag{2. 1}$$

where H_I is the entropy of the input per degree of freedom and H_0 is that of the output.

This relation has been derived from the formula

$$f_0(t) = \int_0^\infty f_I(t-\tau)\, dK(\tau) \,, \tag{2. 2}$$

where $f_I(t)$ and $f_0(t)$ are input and output signal respectively. Fourier transform of (2.2) gives

$$A_0(\omega) = A_I(\omega)\, k(\omega) \,. \tag{2. 3}$$

$k(\omega)$ is given by

$$\int_0^\infty e^{-i\omega t} dK(t) = k(\omega) \,.$$