99. *On the Classification of Symmetric Fuchsian Groups of Genus Zero*

By Tadashi KURODA
Mathematical Institute, Nagoya University, Nagoya

(Comm. by K. KUNUGI, M.J.A., Oct. 12, 1953)

1. Let $\{\alpha_i\}$ $(i=0,1,2,\ldots)$ be a finite or an enumerable number of circular open arcs in the unit circle $|z|<1$ which are orthogonal to the circumference $|z|=1$ and disjoint each others in $|z|<1$ and let D_0 be the simply connected domain in $|z|<1$ bounded by $\{\alpha_i\}$ $(i=0,1,2,\ldots)$ and the closed set E on $|z|=1$. If $\tilde{D}_0$ is the reflection of D_0 with respect to an arc of $\{\alpha_i\}$, say α_0, then the domain $D_0 \cup \alpha_0 \cup \tilde{D}_0$ is a fundamental domain of a symmetric Fuchsian or fuchsoid group $\mathfrak{G}$ without any elliptic transformation and of genus zero. Conversely, such a group has a fundamental domain as stated above.

We denote by $\{\tilde{\alpha}_i\}$ $(i=0,1,2,\ldots)$ the boundary arcs of D_0 corresponding to $\{\alpha_i\}$ $(i=0,1,2,\ldots)$ by $\mathfrak{G}$. $\tilde{\alpha}_0$ is identical to α_0. Identifying the equivalent points on α_i and $\tilde{\alpha}_i$ $(i=1,2,\ldots)$, we get an open Riemann surface $\hat{D}$. This surface $\hat{D}$ can be decomposed by a relative boundary C into two parts D and $\tilde{D}$, each one of which is the image of the other by an indirectly conformal mapping. And $D\cup C$ (or $\tilde{D}\cup C$) is comformally equivalent to $D_0 \cup \bigcup_{i=0}^{\infty} \alpha_i$ (or $\tilde{D}_0 \cup \bigcup_{i=0}^{\infty} \tilde{\alpha}_i$).

2. We state here some notations. Let HB or HD be the class of single-valued harmonic functions bounded or Dirichlet bounded in a region. If there exists no non-constant function of HB (or HD) in D_0 which equals to zero on $\Gamma = \bigcup_{i=0}^{\infty} \alpha_i$, then we may say that D_0 belongs to the class SO_{HB} (or SO_{HD}). If any function of HB in D_0, whose normal derivative vanishes at every point on Γ, reduces to a constant, we say that D_0 belongs to the class NO_{HB}.

Further we denote by O_G the class of Riemann surfaces with null boundary and by O_{AB} (or O_{AD}) the class of Riemann surfaces on each of which there exists no non-constant single-valued bounded (or Dirichlet bounded) analytic function.

3. Ullemar (=Uskila[7)8)]) classified the symmetric Fuchsian or fuchsoid groups $\mathfrak{G}$ without any elliptic transformation and of genus zero according to the existence of a certain kind of automorphic functions for $\mathfrak{G}$. More precisely, $\mathfrak{G}$ belongs to positive type or null type with respect to bounded (or Dirichlet bounded) automorphic functions according to whether in $|z|<1$ there exists a non-