95. *Note on the Fibering of an (n−1)-connected Space by Spheres*

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§ 1. The main object of this note is to study an $(n-1)$-connected space, whose fibering by spheres is impossible. This is somewhat concerned with a problem of D. Montgomery and H. Samelson[1], on the fibering of a Euclidean space by compact fibres.

§ 2. Let X be an $(n-1)$-connected space $(n>2)$; namely it satisfies the conditions $\pi_i(X)=0$ $(i=0,1,\dots,n-1)$.[2] We shall also assume that X is a fibre bundle[3], whose fibre is a $(k-1)$-sphere S^{k-1} $(n>k>1)$; and let us denote whose base space as Y. Now, we shall denote following J. H. C. Whitehead[4], with ΔY the minimum dimensionality of all CW-complexes which dominate Y.[5]

If $p: X\to Y$ is the projection, we obtain an exact sequence of the homotopy groups

$$\begin{array}{ccccccc}\cdots\to\pi_i(X)\to & \pi_i(X,S_0^{k-1}) & \overset{\partial}{\to} & \pi_{i-1}(S_0^{k-1}) & \to \pi_{i-1}(X)\to\cdots \\ & \downarrow p_* & & & \\ & \pi_i(Y), & & & \end{array} \tag{1}$$

where ∂ is the boundary operator, p_* is the isomorphism induced by p, and S_0^{k-1} is a fixed fibre oriented suitably. From the exactness of (1) and from the assumption on X, we obtain the following isomorphism onto:

$$\partial\, p_*^{-1}: \pi_i(Y)\to\pi_{i-1}(S_0^{k-1}) \quad (2\leqq i\leqq n-1).$$

Next, let E^k be an n-dimensional oriented cell, and S^{k-1} be the boundary sphere of E^k oriented coherently with E^k; let $f: S^{k-1}\to S_0^{k-1}$ be a homeomorphism of degree $+1$. Let $g: (E^k, S^{k-1})\to(S^k, s_0)$ be a mapping onto a k-dimensional oriented sphere S^k such that $g\mid \mathrm{Int}\, E^k$ is a homeomorphism of degree $+1$, and $g(S^{k-1})=s_0$, where s_0 is a fixed point on S^k. From these mappings and from (1), we obtain the following diagramm:

$$\begin{array}{ccccc} \pi_i(S^k) & \xleftarrow{g_*} & \pi_i(E^k, S^{k-1}) & \xrightarrow{\partial'} & \pi_{i-1}(S^{k-1}) \\ & & & & \downarrow f_* \\ \pi_i(Y) & \xleftarrow{p_*} & \pi_i(X, S_0^{k-1}) & \xrightarrow{\partial} & \pi_{i-1}(S_0^{k-1}). \end{array} \tag{2}$$

Here, ∂' is the boundary operator, which is an isomorphism onto for all $i\geqq 2$; f_* and g_* are homomorphisms induced by f and g