217. *A Class of Markov Processes with Interactions.* II

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Here, we look at the branches which describe the interactions between particles of the model in [4]. This leads to finer proofs of Chapman-Kolmogorov equation and the backward equation. A consistency condition holds for probabilities of events which are determined by bundles of these branches.

1. To consider the simplest model with binary interactions, let $q(t, y) \equiv q_1(t, y)$ and $q_0 \equiv q_2 \equiv q_3 \equiv \cdots \equiv 0$, and write $\pi(y' \mid t, y, E)$ for $\pi_1(y_1 \mid t, y, E)$ in **1** of [4].[1] Then, the forward and the backward equations are

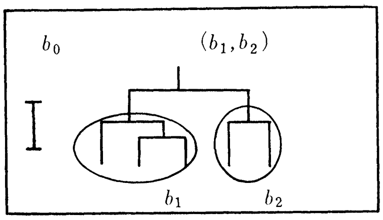
$$P^{(f)}(s, x, t, E) = P_0(s, x, t, E) + \int_s^t d\tau \int_{R^2} P^{(f)}(s, x, \tau, dy) \times P^{(f)}_{s,\tau}(dy') q(\tau, y) \int_R \pi(y' \mid \tau, y, dz) P_0(\tau, z, t, E), \tag{1}$$

$$P^{(P^{(f)}_{s_0 s})}(s, x, t, E) = P_0(s, x, t, E) + \int_s^t d\tau \int_{R^2} P_0(s, x, \tau, dy) \times P^{(f)}_{s_0 \tau}(dy') q(\tau, y) \int_R \pi(y' \mid \tau, y, dz) P^{(P^{(f)}_{s_0 \tau})}(\tau, z, t, E), \tag{2}$$

where $P^{(f)}_{s,\tau}(E) \doteq \int_R f(dx) P^{(f)}(s, x, \tau, E), \quad s_0 \leq s \leq t.$

Let T be the set of all branches which grow downward with binary branching points and the trivial branch (or a pole) b_0. For b_1 and b_2 in T, $b = (b_1, b_2)$ is the branch which has b_1 and b_2 on the left and the right side of the highest branching point. Length $l(b)$ and the number of the end points $\sharp(b)$ are defined by

$$l(b_0) = 0, \; l((b_1, b_2)) = 1 + \max(l(b_1), l(b_2)),$$
$$\sharp(b_0) = 1, \; \sharp((b_1, b_2)) = \sharp(b_1) + \sharp(b_2).$$



When $\sharp(b) = n$, let $b(b_1, \cdots, b_n)$ be the branch b with branches $b_1, \cdots, b_n$ connected at the end points, with b_k at the k-th end point from the left. We write $b \geq b'$ when $b = b'(b_1, \cdots, b_n)$. Since the branches $b_1, \cdots, b_n$ are determined

1) This is for the simplicity of descriptions. Results in this paper can be extended to the models in [4].