182. *Local Comparison Theorems for a Certain Class of Multi-Dimensional Markov Processes of Transient Type*

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Our aim of this paper is to give certain local comparison theorems about hitting probability and fine topology in connection with Green functions. In the author's preceding papers [1], [2] and [3] we have given some comparison theorems on fine topology in case Markov processes have Green functions with a certain kind of isotropic singularities. Using the following results of this paper we can get a local comparison theorem about fine topology without the assumption on the singularity of Green functions, or without assuming the existence of Green functions. But our result may not contain the result of Theorem 5 in [2] completely if we restrict our processes to those in Theorem 5 in [2].

1. Notations and definitions. Without special mentioning the process $X=(x_t, \zeta, M_t, P_x)$ we treat is assumed to be a standard process on a domain $\Omega\subset R^d$ (d-dimentional Euclidean space) such that each point in Ω can not be reached[1] and G^{α}[2] maps $C_K(\Omega)$ into $C_0(\Omega)$ for some $\alpha>0$, where $C_0(\Omega)$ denotes the set of continuous functions on Ω vanishing at infinity and $C_K(\Omega)$ is the set of functions $\in C_0(\Omega)$ with compact support in Ω, and $\int_0^{+\infty} T_t f dt$ is finite for each $f\in C_K(\Omega)$.

Definition 1. A non-negative, measurable kernel $G(x, y)$ on $\Omega\times\Omega$ is said to be a regular Green function of the process X, if it satisfies the following conditions.

i) $\int_0^{+\infty} T_t f(x)dt=\int_\Omega G(x, y)f(y)dy$ for each $f\in C_K(\Omega)$.

ii) $G(x, y)$ is bounded except at each neighborhood of the diagonal on $\Omega\times\Omega$.

iii) $\lim_{n\to+\infty} \inf_{\substack{x\in Q_n\\ y\in Q_n}} G(x, y)=+\infty$ for each ball decreasing to a point.

iv) For each compact or open set K with compact closure in Ω, there exists a measure $\mu_K(dy)$ supporting on $\bar{K}$ such that

$$P_x(\sigma_K<+\infty)=\int G(x, y)\mu_K(dy).$$

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1) $P_x(\sigma\{x_0\}<+\infty)=0$ for each point $x, x_0\in\Omega$, where $\sigma_B=\inf(t>0, x_t\in B)$.

2) $G^\alpha f(x)=\int_0^{+\infty} e^{-\alpha t}T_t f(x)dt$ for $f\in C_K(\Omega)$.