181. *Continuity of Stochastic Processes on Metric Spaces*

By Takayuki KAWADA
Kobe College of Commerce

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1970)

1. After A. N. Kolmogorov had presented the continuity condition of stochastic processes ([5]), several generalizations have been considered (e.g. [1]-[4]). But H. Cramér's idea in [1] permits us to obtain the continuity conditions in the more general situations; Let $\{x(t,\omega); t\in S\}$ be stochastic processes, based on a probability space $(\Omega, \mathcal{B}, P)$, of which parameter t runs over a compact metric space (S, d), and of which value is taken in a complete metric space (M, r). Here their metrics are d and r, respectively. Denote by $N(\varepsilon)$ the minimal number of ε-net of the space S.*) Then we establish the followings:

Theorem 1. *Suppose that*

$$(1)\qquad P[r(x(t), x(s))\geq g(d(t,s))]\leq q(d(t,s)),$$

where $g(h)$ and $q(h)$ are even, non-decreasing functions in $h>0$, and that

$$(2)\qquad \sum_{n=1}^{\infty} g(2^{-n})<\infty, \qquad \sum_{n=1}^{\infty} N^2(2^{-n-1})\cdot q(2^{-n+2})<\infty.$$

Then the stochastic processes have continuous version.

Theorem 2. *Suppose* (1) *above and that*

$$(3)\qquad \sum_{k=1}^{\infty} g(2^{-n-k})<C\cdot g(2^{-n}), \qquad \sum_{n=1}^{\infty} N^2(2^{-n-1})\cdot q(2^{-n+2})<\infty,$$

and

$$(4)\qquad g(4h)<C'\cdot g(h) \qquad \textit{for sufficiently small } h,$$

where n is any positive integer, and C and C' are some positive constants. Then the stochastic processes have g-Hölder continuous version.

2. By A_n, we denote the elements of 2^{-n}-*net*; $A_n=\{t_n^k; k=1,2,\cdots,N(2^{-n})\}$, $n=1,2,3,\cdots$, and we set $D=\bigcup_{n=1}^{\infty} A_n$. By F, we define the space of all M-valued, non-random functions, and by F_n the elements of F such that

$$F_n=\{f(t); r(f(x), f(y))\leqq g(d(x,y)),$$
$$\text{for}\quad (x,y), x\in A_n, y\in A_{n+1} \text{ and } d(x,y)\leqq 2^{-n+2}\},$$

where $g(h)$ is one cited in (1). Further, we set $U_n=\bigcap_{j=n}^{\infty} F_j$, and

---

*) $\log N(\varepsilon)$ is called ε-entropy of the space S.