180. *Complex Powers of Non-elliptic Operators*

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1. Introduction.

In the present paper we shall construct symbols of pseudo-differential operators which define complex powers of a pseudo-differential operator in a class S_λ^m which contains semi-elliptic operators. Complex powers of an elliptic operator as pseudo-differential operators are defined by Burak [1] and Seely [4]. They constructed symbols through Dunford's integrals for an elliptic operator defined on a C^∞ compact manifold without boundary, so the global ellipticity of the operator is required. Here, we shall construct symbols only by local calculation. The precise calculation of symbols for iterations of a pseudo-differential operator gives the relations among polynomials in coefficients of the symbols, then the symbols of integral powers of an operator is extended to be those of complex ones.

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2. Definitions and lemmas.

Definition 1. *A real valued $C^\infty(R^n)$ function $\lambda(\xi)$ is called a basic weight function when it satisfies the conditions*:

(2.1) $$1\leqq\lambda(\xi)\leqq A(1+|\xi|),$$

(2.2) $$|\partial_\xi^\alpha\lambda(\xi)|\leqq A_\alpha\lambda(\xi)^{1-|\alpha|} \quad \textit{for any} \quad \alpha,$$

for some constants A and A_α. (*See Kumano-go* [3].)

Definition 2. *Let $\lambda(\xi)$ be a basic weight function. Then we say $p(x,\xi)\in S_\lambda^m$, when $p(x,\xi)\in C^\infty(R^n\times R^n)$ and*

(2.3) $$|D_x^\alpha\partial_\xi^\beta p(x,\xi)|\leqq C_{\alpha,\beta}\lambda(\xi)^{m-|\beta|} \quad \textit{for any} \quad \alpha,\beta,$$

for some constants $C_{\alpha,\beta}$, where $D_x=(-i)\partial_x$.

For $p(x,\xi)\in S_\lambda^m$ we define the pseudo-differential operator $p(X,D_x)$ by

(2.4) $$p(X,D_x)u(x)=\frac{1}{(2\pi)^n}\int e^{ix\cdot\xi}p(x,\xi)\hat{u}(\xi)d\xi,$$

where $u(x)$ is a C^∞ function which together with all their derivatives decreases faster than any powers of $|x|$ as $|x|\to\infty$, and

$$\hat{u}(\xi)=\int e^{-ix\cdot\xi}u(x)dx.$$

We denote the symbol of an operator $p(X,D_x)$ by

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