179. *On Some Invariant Subspaces*

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(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1970)

Let X be a compact Hausdorff space and let A be a function algebra on X. Throughout this paper, ϕ will be a fixed multiplicative linear functional on A which admits a unique representing measure m. Further we assume that the Gleason part of ϕ is non trivial. We denote by A_0 the maximal ideal associated with ϕ; $A_0=\{f\in A:\phi(f)=0\}$. Let $H^2=H^2(dm)$ be the closure $[A]_2$ of A in $L^2=L^2(dm)$. We put $H_0^2=\left\{f\in H^2;\int f dm=0\right\}$. We shall refer to Browder [1] for the abstract function theory in this situation.

Let M be a closed subspace of H^2. M is called simply invariant if $[A_0M]_2 \subsetneq M$. We call M complementary invariant if $H^2\ominus M$, the orthogonal complement of M in H^2, is simply invariant. The purpose of this paper is a characterization of the complementary invariant subspace.

It is well known that L^2 admits the orthogonal decomposition $L^2=H^2\oplus\bar{H}_0^2$, where the bar denotes the complex conjugation. We denote by P the orthogonal projection of L^2 onto H^2 As Wermer has shown, there exists an inner function Z such that $H_0^2=ZH^2$. (See [1] Lemma 4.4.3 for our situation.) We define the backward shift operator T on H^2 by

$$Tf=\frac{f-\int f dm}{Z} \quad (f\in H^2).$$

Theorem. *The complementary invariant subspaces of H^2 are precisely the subspaces of the form*

$$P[Tq\cdot\bar{H}^2],$$

where q is an inner function. q is determined by the subspace up to a constant factor.

Proof. Let M be a complementary invariant subspace of H^2. Then $N=H^2\ominus M$ is a simply invariant subspace of H^2. Therefore, by the generalized Beurling theorem (for instance, see [1] Theorem 4.3.5), N has the form $N=qH^2$, where q is inner. For simplicity, we put $h=Tq$. Evidently $h\in L^\infty\cap H^2$. Since $\int Z dm=0$ and q is inner, we have