177. *Topological Groups and the Generalized Continuum Hypothesis*

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The purpose of this Note is to show that the proposition (I) and (II) below are equivalent to the Generalized Continuum Hypothesis, in the Zermelo-Fraenkel set theory, without the Axiom of Choice. Propositions (I) and (II) deal with a topological property—property K —of a particular type of topological group. Property K is related with the uniform continuity of all continuous real-valued functions defined on a group.

1. Preliminaries and notations.

We consider the Zermelo-Fraenkel axiomatic set theory, without the Axiom of Choice.

For any set $Z, 2^Z$ is the potence set of Z and $2Z$ is the set $\{0,1\}\times Z$. In §3, according to the notations of [2], for any set Z, we put $P_0(Z)=Z$, $P(Z)=2^Z$ and $P_i(Z)=P(P_{i-1}(Z))$, $i=1,2,3,4$.

For any two sets A and $B, A\leqslant B$ means that there is an injective map from A into B; $A<B$ means that $A\leqslant B$ and A and B are not equipotent sets; $A\approx B$ means that A and B are equipotent sets (i.e., by virtue of Bernstein-Cantor theorem, $A\leqslant B$ and $B\leqslant A$). Finally, $A+B$ indicates the disjoint sum of the sets A and B.

Let (G,τ) be a topological group and let U denote the right uniformity of G. (G,τ) *has property* K if and only if any continuous real-valued function on G is a uniformly continuous map of (G,U) into R (i.e., if $f:G\to R$ is continuous and r is a positive real number, there is an open neighborhood of the neutral element of G, V, such that if $x,y\in G$ and $y\in Vx$, then $|f(x)-f(y)|<r$). The group operation is denoted multiplicatively.

Let S be an infinite set. $\{0,1\}^S$ is an algebraic group with the following operation: if $x=(x_s)_{s\in S}$ and $y=(y_s)_{s\in S}$ belong to $\{0,1\}^S$, then $xy=(x_sy_s)_{s\in S}$, where $01=10=1$ and $00=11=0$.

Let M be an infinite set, S be a set with $2^M\leqslant S$, and put $G(S)=\{x\in\{0,1\}^S \mid \{s\in S\mid x_s=1\}\leqslant M\}$, where $x=(x_s)_{s\in S}$. We say that $G(S)$ is a group if $G(S)$ is a subgroup of $\{0,1\}^S$.

For any infinite sets M, Y and S, with $M<Y\leqslant 2^M\leqslant S$, let $B(Y)$ be the set of all elements $G(S)\cap\prod_{s\in S}V_s$, where $V_s\subset\{0,1\}$, $\forall s\in S$ and