176. *On the Williamson's Conjecture*

By Tetsuhiro SHIMIZU
Department of Mathematics, Tokyo Institute of Technology

(Comm. by Kinjirô KUNUGI, M. J. A., Sept. 12, 1970)

1. Let G be a non-discrete locally compact abelian group with the dual group Γ of G. We will denote by $M(G)$ the Banach algebra of all bounded regular Borel measures on G under the convolution multiplication.

It is known that there exists a compact commutative topological semigroup S and an order preserving isometric-isomorphism θ of $M(G)$ into $M(S)$ such that:

(a) if $\mu \in M(G)$, $\nu \in M(S)$ and ν is absolutely continuous with respect to $\theta\mu$, then there is $\omega \in M(G)$ such that $\theta\omega=\nu$: and

(b) each multiplicative linear functional h on $M(G)$ has the form

$$h(\mu)=\int_S f d\theta\mu \qquad (\mu \in M(G))$$

for an unique nonzero continuous semicharacter f on S (cf. [4]).

The set of all nonzero continuous semicharacters on S is denoted by $\hat{S}$. We may consider $\hat{S}$ to be the maximal ideal space of $M(G)$. Furthermore, $\hat{S}$ is a compact semigroup and Γ may be considered to be the maximal group at the identity of $\hat{S}$ (cf. [5]).

We denote by $\varDelta$ the subset of $\hat{S}$ consisting of functionals symmetric in the sense that $\hat{\mu}^*(f)=\overline{\hat{\mu}(f)}$ for any $\mu \in M(G)$, where * denotes the usual involution on $M(G)$. Let $M(\varDelta)=\{\mu \in M(G): \hat{\mu}(f)=0$ for all $f \in \hat{S}\backslash\varDelta\}$.

Let $M_c(G)$ denote the algebra of all continuous measures of $M(G)$.

Our purpose is to show that the following *Williamson's conjecture* (cf. [6]) is true.

Williamson's conjecture: *If $\mu \in M(\varDelta)$, then $\mu \in M_c(G)$.*

2. By (a), if $f \in \hat{S}$ and $\mu \in M(G)$, then there is a measure $\mu_f \in M(G)$ such that $d\theta\mu_f=fd\theta\mu$.

The following lemmas are essential to prove that Williamson's conjecture is true.

Lemma 1. *If $f \in \Gamma$ and $\mu \in M(G)$, then $d\theta\mu_f^*=fd\theta\mu^*$.*

Lemma 2. *If $f \in \Gamma$ and $g \in \hat{S}\backslash\varDelta$, then $fg \in \hat{S}\backslash\varDelta$.*

For any $f \in \hat{S}$, let $S_f=\{s \in S: f(s)\neq 0\}$ and let $J_f=\{s \in S: f(s)=0\}$.

Theorem 3. *If $g \in \hat{S}\backslash\varDelta$ and $\mu \in M(\varDelta)$, then $\theta\mu|_{S_g}$, the restriction to S_g of $\theta\mu$, is zero measure. In particular, $M(\varDelta)\subset M_c(G)$.*

From this, it follows that: