205. *On Potent Rings. III*

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In [5], [6], we have mainly investigated potent irreducible rings. The purpose of this paper is to prove that a right locally uniform potent ring with zero right singular ideal is an essential irredundant subdirect sum PI-rings and conversely. A number of concepts and results are needed from [5] and [6].

By the same argument as in Theorem 2.2 of [2], we obtain the following

Proposition 1. *Let R be a right locally uniform ring with $Z_r(R)=0$, let I be a right ideal of R and let I^* be a unique maximal essential extension of I in R. Then $I^*=\{a \in R \mid aE \subseteqq I$ for some $E \subset' R\}$.*

Let R be a right locally uniform ring with $Z_r(R)=0$ and let $\hat{R}$ be the maximal right quotient ring of R. Then the mappings

$$A \to E_R(A),\ A \in L_r^*(R);\ \hat{A} \to \hat{A} \cap R,\ \hat{A} \in L_r^*(\hat{R})$$

are mutually inverse isomorphisms between $L_r^*(R)$ and $L_r^*(\hat{R})$, where $E_R(A)$ is a right R-injective hull of A (see [1]). Let A be an element of $L_r^*(R)$. Then we denote by $\hat{A}$ the element of $L_r^*(\hat{R})$ which corresponds to A. Clearly $\hat{A}$ is a right R-injective hull of A and is right $\hat{R}$-injective. Let A and B be uniform right ideals of R. As in [5], A and B are similar (in symbol; $A \sim B$) iff A and B contain mutually isomorphic nonzero right ideals A' and B', respectively. The set of all uniform right ideals of R can be classified by the equivalence relation $\sim$. $\{A_i\}$ will denote the class containing the uniform right ideal A_i. We now set $R_i=(\sum_{A \in \{A_i\}} A)^*$. Then we obtain

Proposition 2. *Let R be a right locally uniform ring with $Z_r(R)=0$. Then the following properties hold:*

(1) *$\sum_{A \in \{A_i\}} A$ is a two-sided ideal.*

(2) *R_i is an ideal of R for each i.*

(3) *If B is a uniform right ideal of R and if $B \subseteqq R_i$, then $B \sim A_i$.*

(4) *$\sum_i R_i$ is a direct sum.*

Proof. Let A be a uniform right ideal and let x be an element of R. Then $xA=0$ or $xA \cong A$ and hence (1) follows immediately.

(2) follows immediately from Proposition 1 and (1).

(3) is obtained by the same argument as in Lemma 5.5 of [3].

(4) We can prove that $\hat{R}_i$ is an $\hat{R}$-injective hull of the sum of all minimal right ideals of $\hat{R}$ which are isomorphic to $\hat{A}_i$. Hence the