227. *Wirtinger Presentations of Knot Groups**)

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In this note we shall give an algebraic proof of the following theorem, which is concerned with [2] and [3].

Theorem. *If a finitely presented group G satisfies the conditions* (a) $G/[G,G]$ *is isomorphic to a free abelian group of rank* $\mu\geqq 1$, (b) *the weight of G equals to* μ, (c) $H_2(G)=0$, *then G has Wirtinger presentations.*

1. Let E be an arbitrary subset of a group G. We shall denote by E^G the normal closure of E. If $E^G=G$ for some finite subset $E=(g_1,\cdots,g_n)$, then we shall call E a *nucleus* of G, and call n the order of the nucleus. Kervaire [2] called the minimal order of nuclei of G the weight of G.

The following proposition is obvious.

(1.1) Let $(g_1,\cdots,g_n)$ and $(h_1,\cdots,h_n)$ be n-tupels of G such that the transformation $(g_1,\cdots,g_n)\to(h_1,\cdots,h_n)$ is obtained by a finite sequence of transformations of the following types:

(i) $(g_1,\cdots,g_n)\to(g_1^{\varepsilon_1},\cdots,g_n^{\varepsilon_n}),\ \varepsilon_i=\pm 1,\ i=1,\cdots,n,$

(ii) $(g_1,\cdots,g_n)\to(g_{i_1},\cdots,g_{i_n})$, where $(i_1,\cdots,i_n)$ is a permutation of $(1,2,\cdots,n)$,

(iii) $(\cdots,g_i,\cdots,g_j,\cdots)\to(\cdots,g_i,\cdots,g_i^{\varepsilon}g_j,\cdots)$ or $(\cdots,g_i,\cdots,g_jg_i^{\varepsilon},\cdots),\ \varepsilon=\pm 1$. Then $(h_1,\cdots,h_n)^G=(g_1,\cdots,g_n)^G$.

Let $(x_1,\cdots,x_n:r_1,\cdots,r_m)$ be a presentation of a group G. If each relator r_i is described in a form $x_i^{-1}w_{ij}x_jw_{ij}^{-1}$, i.e. $x_i=w_{ij}x_jw_{ij}^{-1}$ as a relation, then we call the presentation a *Wirtinger presentation* of G.

Let $F=F[x_1,\cdots,x_n]$ be a free group generated by free generators $x_1,\cdots,x_n$, and let R be the kernel $(r_1,\cdots,r_m)^F$ of the homomorphism $\varphi:F\to G$. Hopf [1] defined the second homology group $H_2(G)$ as the group $[F,F]\cap R/[F,R]$, and proved that it does not depend on the underlying free group F.

(1.2) Suppose a group G satisfies the condition (c) of the theorem and $(x_1,\cdots,x_n:r_1,\cdots,r_l,r_{l+1},\cdots,r_m)$ is a presentation of G. If $r_{l+1},\cdots,r_m\in[F,F]$, then G has also a presentation $G=(x_1,\cdots,x_n:r_1,\cdots,r_l,[r_i,x_j],i=l+1,\cdots,m,j=1,\cdots,n)$.

Proof. We shall prove that $(r_1,\cdots,r_m)^F=(r_1,\cdots,r_l,\{[r_i,x_j]\})^F$. $(r_1,\cdots,r_m)^F\supset(r_1,\cdots,r_l,\{[r_i,x_j]\})^F$ is trivial. Since $r_k\in[F,F]$ for k

*) Dedicated to Professor Keizo Asano on his 60th birthday.