28. *Angular Cluster Sets and Horocyclic Angular Cluster Sets*

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1. In [1] Bagemihl began a study of relations between non-tangential (angular) boundary behaviors and horocyclic boundary behaviors of meromorphic functions defined in the open unit disk D of the complex plane. This study has been continued by Dragosh in [2] and [3]. The purpose of the present paper is to sharpen some of results of these investigations by the method of Dolzhenko's paper.

Notation and definitions. Unless otherwise stated, $f: D \to W$ shall mean $f(z)$ is an arbitrary function (generally not unique) defined in the open unit disk $D: |z|<1$ and assuming values in the extended complex plane W. The unit circle $|z|=1$ is denoted by Γ.

A circle internally tangent to Γ at a point $\zeta \in \Gamma$ is called a horocycle at ζ, and will be denoted by $h_r(\zeta)$, where r $(0<r<1)$ is the radius of the horocycle.

Given a horocycle $h_r(\zeta)$ at a point $\zeta \in \Gamma$, the region interior to $h_r(\zeta)$ is called an oricycle at ζ, and will be denoted by $K_r(\zeta)$, or simply $K(\zeta)$ without specifying r. The half of $K_r(\zeta)$ lying to the right of the radius at ζ as viewed from the origin will be denoted by $K_r^+(\zeta)$ and $K_r^-(\zeta)$ denotes the left half of $K_r(\zeta)$ analogously.

Suppose that $0<r_1<r_2<1$. Let $r_3(0<r_3<1)$ be so large that the circle $|z|=r_3$ intersects both of the horocycles $h_{r_1}(\zeta)$ and $h_{r_2}(\zeta)$. We define the right horocyclic angle $H^+_{r_1,r_2,r_3}(\zeta)$ at ζ with radii r_1, r_2, r_3 to be

$$H^+_{r_1,r_2,r_3}(\zeta)=\text{com}\,(\overline{K^+_{r_1}(\zeta)}) \cap K^+_{r_2}(\zeta) \cap \{z: |z| \geqq r_3\},$$

where the bar denotes closure and comp denotes complement, both relative to the plane. The corresponding left horocyclic angle is denoted $H^-_{r_1,r_2,r_3}(\zeta)$. We write $H_{r_1,r_2,r_3}(\zeta)$ to denote a hyrocyclic angle at ζ without specifying whether it be right or left, or simply $H(\zeta)$ in the event r_1, r_2, r_3 are arbitrary.

We assume the reader to be familiar with the rudiments of the cluster sets.

$C_V(f,\zeta)$, the angular cluster set of $f(z)$ at ζ on a Stolz angle $V(\zeta)$;

$C_K(f,\zeta)$, the oricyclic cluster set of $f(z)$ at ζ on an oricycle $K(\zeta)$;

$C_H(f,\zeta)$, the horocyclic angular cluster set of $f(z)$ at ζ on a horocyclic angle $H(\zeta)$.

A point $\zeta \in \Gamma$ is said to be a horocyclic angular Plessner point