27. *Localization Theorem in Hyperbolic Mixed Problems*

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(Comm. by Kinjirô KUNUGI, M. J. A., Jan. 12, 1971)

Introduction. Recently Atiyah, Bott and Gårding [1] have studied some interesting properties (behavior near the wave fronts, supports, singular supports and lacunas, etc.) and structures of fundamental solutions of hyperbolic differential operators with constant coefficients. It seems that some of their methods can be applicable to the study of Riemann or Green's functions (kernels) for hyperbolic mixed problems in a quarter-space. The properties of such Riemann functions are less investigated. For example there are Deakin [2], Duff [3] Hersh [4], etc. In this note we present one of properties which can be easily proved, more precisely "localization theorem" corresponding to one in the free space-time case. The idea of localizing fundamental solutions is due to Hörmander [6].

1. Riemann or Green's functions. Let $\boldsymbol{R}^n$ denote the n-dimensional euclidean space and $\boldsymbol{\Xi}^n$ its complex dual space, we shall write $x'=(x_1, \cdots, x_{n-1})$, $x''=(x_2, \cdots, x_n)$ for the coordinate $x=(x_1, \cdots, x_n)$ in $\boldsymbol{R}^n$ and $\xi'=(\xi_1, \cdots, \xi_{n-1})$, $\xi''=(\xi_2, \cdots, \xi_n)$ for the dual coordinate $\xi=(\xi_1, \cdots, \xi_n)$. The variable x_1 will play the role of "time", the variables $x_2, \cdots, x_n$ will play the role of "space". We shall also denote by $\boldsymbol{R}^n_+$ the half-space $\{x=(x', x_n) \in \boldsymbol{R}^n, x_n>0\}$. For differentiation we will use the symbol $D=1/i\cdot\partial/\partial x$.

Let $P=P(\xi)$ be a hyperbolic polynomial of n variables ξ with respect to $\vartheta=(1, 0, \cdots, 0) \in \mathrm{Re}\, \boldsymbol{\Xi}^n$ in the sense of Gårding, i.e. $P_m(\vartheta)\neq 0$ and $P(\xi+t\vartheta)\neq 0$ when ξ is real and $\mathrm{Im}\, t$ is less than some fixed number where P_m denotes the principal part of P. We consider the following mixed initial-boundary value problem for the hyperbolic operator $P(D)$ in a quarter-space

$$(1) \qquad P(D)u(x)=f(x), \quad x \in \boldsymbol{R}^n_+, \quad x_1>0,$$

$$(2) \qquad (D_1^k u)(0, x'')=0, \quad k=0, 1, \cdots, m-1, \quad x_n>0,$$

$$(3) \qquad B_j(D)u(x)\Big|_{x_n=0}=0, \quad j=1, \cdots, l, \quad x_1>0.$$

Here $B_j(D)$ are boundary operators with order m_j. The number of boundary conditions will be determined later on.

We assume that the coefficients of ξ_n^m in $P(\xi)$ and $\xi_n^{m_j}$ in $B_j(\xi)$ are different from zero, i.e. that the hyperplane $x_n=0$ is non-characteristic for $P(D)$ and $B_j(D)$. We shall construct the Riemann function $G(x, y)$ which describes the propagation of waves produced by a unit impulse