54. *Functional Dimension of Tensor Product*

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§1. **Introduction.** The purpose of this paper is to give a proof to the fact that the functional dimension of the tensor product of two topological vetcor spaces is equal to the sum of their functional dimensions.

A. N. Kolmogorov [1] showed that the asymptotic behavior of number of elements of a minimal ε-net of a totally bounded subset in a topological vector space plays the role of dimension of the space. He [2] also introduced the notions of the approximative dimension and the functional dimension of topological vector spaces. The functional dimension is not trivial for σ-Hilbert nuclear spaces as is shown in I. M. Gel'fand's book [3].

In this paper we modify the definition of the functional dimension d_f of σ-Hilbert nuclear spaces to the number which is equal to the functional dimension (defined by Kolmogorov) minus 1, and we prove the following theorem:

Theorem. *Let E_1 and E_2 be σ-Hilbert nuclear spaces. Then*
$$d_f(E_1 \otimes E_2) = d_f(E_1) + d_f(E_2).$$

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§2. **Notations.** We follow notations used by Kolmogorov [4]. Let E be a topological vector space, K be a totally bounded subset of E and S be its convex absorbing and barrelled neighbourhood of 0 in E. Then we call *ε-entropy* $H_\varepsilon(S, K)$ of K (with respect to S) the infimum of logarithm of number of *ε-nets* of K (with respect to S); that is,
$$H_\varepsilon(S, K) = \inf \{\log (\sharp N) ; N \subset E, \forall k \in K, \exists n \in N, k \in n + \varepsilon S\}.$$

We use the following notations for infinitesimals: $f(x) \succcurlyeq g(x)$ means $\lim_{x\to\infty} g(x)/f(x) < +\infty$; $f(x) \asymp g(x)$ means $f(x) \preccurlyeq g(x)$ and $f(x) \succcurlyeq g(x)$; $f(x) = \Omega(g(x))$ means $\lim_{x\to\infty} (f(x))^n / g(x) = 0$.

In this paper the notation log stands for the logarithm with respect to the base 2.

§3. **Theorem of Mityagin and σ-Hilbert nuclear spaces.** We define as follows: The set $\mathcal{E}$ is called *$\{a_n\}$-ellipsoid* when $\mathcal{E} = \{(\xi_n) \in (l^2) ; \sum_n |\xi_n a_n|^2 \leqq 1\}$, where $\{a_n\}$ is a monotonous increasing series of such numbers a_n that $a_n \geqq 1$ and $\lim_{n\to\infty} a_n = \infty$; the function $m(t)$ is defined by the formula $m(t) = \sup \{n ; a_n \leqq t\}$; let S be the unit ball in (l^2).