144. On the Global Existence of Real Analytic Solutions of Linear Differential Equations. II

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§0. In our previous note [9] we have presented a global existence theorem of real analytic solutions for a linear differential operator $P(D)$ with constant coefficients assuming condition (1) below.

(1) The principal symbol $P_m(\xi)$ of $P(D)$ is real and of simple characteristics, i.e., $\mathrm{grad}_\xi P_m \neq 0$ whenever $P_m(\xi)=0$, where ξ is a non-zero real cotangent vector.

That is, denoting by $\mathcal{A}(\Omega)$ the space of real analytic functions on $\Omega \subset \boldsymbol{R}^n$, we have obtained a real analytic solution $u(x)$ of $P(D)u(x)=f(x)$ for any $f(x)$ belonging to $\mathcal{A}(\Omega)$ under some geometrical conditions on Ω. (Kawai [9] Theorem 4.)

The purpose of this note is to extend the results of Kawai [9] in two ways, i.e., in §1 we treat differential operators with constant coefficients not necessarily satisfying condition (1) and in §2 we treat strictly hyperbolic operators with real analytic coefficients defined on a real analytic manifold.

In this note we use the same notations as in our previous note [9] and do not repeat their definitions if there is no fear of confusions.

The details and complete arguments will be given somewhere else.

§1. In this paragraph we use the notion of "localization of differential operators with constant coefficients", which is due to Atiyah, Bott and Gårding [2]. Using the notion of localization Andersson [1] introduced the notion of locally hyperbolic operators and investigated the (analytic) singular support of their elementary solutions. (Andersson [1] Definition 3.2. Such operators are considered also in Kawai [11] independently.) In the sequel we follow Andersson [1] and Atiyah, Bott and Gårding [2] in terminology and notations and do not repeat the definitions: roughly speaking a locally hyperbolic operator with constant coefficients is a differential operator whose localization $P_{\xi_0}(D)$ is hyperbolic with respect to some direction $v(\xi_0)$. The inner core of $P_{\xi_0}(D)$, i.e., the component of $\{\xi \in \boldsymbol{R}^n \mid (P_{\xi_0})_p(\xi) \neq 0\}$ containing $v(\xi_0)$, is denoted by $\Gamma(P_{\xi_0}, v(\xi_0))$ and its dual cone by $K(P_{\xi_0}, v(\xi_0))$, where we denote by $(P_{\xi_0})_p(\xi)$ the principal symbol of $P_{\xi_0}(D)$. We remark that we need not pose any conditions on lower order terms of $P(D)$, which