199. *Certain Convexoid Operators*

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1. Introduction. We call a bounded linear operator T on a complex Hilbert space H, according to [2], *paranormal* if

$$\|T^2x\|\geqq\|Tx\|^2 \tag{1}$$

for every unit vector x in H.

It is easy to verify that any hyponormal* operator is paranormal. In fact if T is hyponormal

$$\|Tx\|^2=(T^*Tx, x)\leqq\|T^*Tx\|\leqq\|T^2x\|$$

for every unit vector x.

It is known that there exists a paranormal but non-hyponormal operator and every power of paranormal operator is again paranormal [2], also paranormal operator is normaloid*) [2] [9] and moreover paranormal operator is compact if some of its powers is compact [5] and that compact paranormal operator is normal [9], and the inverse of a paranormal is also [2] [9].

In [1] T. Ando has given an elegant algebraic characterization of paranormal operator and he has proved several interesting results. Some of them are as follows; a bounded linear operator T is normal if and only if both T and T^* are paranormal and they have the common kernel, and moreover a paranormal operator is normal if some of its power is normal as a generalization of Stampfli's result [12] in the case of hyponormal operator.

Following Halmos [7] the numerical range $W(T)$ is defined as follows:

$$W(T)=\{(Tx, x)\,;\,\|x\|=1\}.$$

An operator T is said to be *convexoid* [7] if

$$\overline{W(T)}=co\,\sigma(T)$$

where $co\,\sigma(T)$ means the convex hull of the spectrum $\sigma(T)$ of T and the $\overline{W(T)}$ means the closure of the set $W(T)$. An operator T is said to be *spectraloid* [7] if

$$w(T)=r(T)$$

or equivalently

$$w(T^n)=(w(T))^n \qquad (n=1, 2, \cdots) \text{ [4]}$$

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*) An operator T is said to be *hyponormal* if $\|Tx\|\geqq\|T^*x\|$ for every vector x and *normaloid* if $\|T^n\|=\|T\|^n$ $(n=1, 2, \cdots)$ [7].