

192. Uniform Spaces of Countably Paracompact Character

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We consider here a class of uniform spaces analogous to the spaces of paracompact character studied by J. Ferrier [1]. We show that these spaces are *cb*-spaces in the sense of Mach [2] and that in normal spaces they are equivalent to countably paracompact spaces. The terminology of [2] is used. As in [1] a family $\{X_a : a \in A\}$ of subspaces of a uniform space X is uniformly discrete if there exists a member V of the uniformity such that $V[X_a] \cap V[X_b] \neq \emptyset$ implies that $a = b$.

Definition 1. A uniform space X is of *countably paracompact character* if every countable open cover of X has a σ -uniformly discrete refinement consisting of generalized co-zero sets.

The following proposition follows from the definition.

Proposition 1. *Every uniform space which is the union of a countable number of uniform spaces of countably paracompact character is of countably paracompact character.*

Theorem 1. *Every uniform space of countably paracompact character is a *cb*-space and therefore countably paracompact.*

Proof. Let \mathcal{W} be an increasing countable open cover of a uniform space of countably paracompact character X . By Theorem 1(e) of [2] it is enough to show that there exists a partition of unity subordinate to \mathcal{W} . Let $\mathcal{U} = \cup \{U_n : n = 1, 2, \dots\}$ be a refinement of \mathcal{W} , where, for each n , $\mathcal{U}_n = \{U_{n,a_n} : a_n \in A_n\}$ is a uniformly discrete family of generalized co-zero sets.

For each n , let V_n be a member of the uniformity on X such that $V_n[U_{n,a_n}] \cap V_n[U_{n,b_n}]$ is not empty implies that $a_n = b_n$. For each pair (n, a_n) , choose $W_{n,a_n} \in \mathcal{W}$ such that $U_{n,a_n} \subset W_{n,a_n}$. Since the generalized co-zero set U_{n,a_n} is contained in $V_n[U_{n,a_n}] \cap W_{n,a_n}$, which is open, we can choose a continuous function $f_{n,a_n} : X \rightarrow [0, 2^{-n}]$ such that $f_{n,a_n}(x) \neq 0$ when $x \in U_{n,a_n}$ and $f_{n,a_n}(x) = 0$ when $x \notin V_n[U_{n,a_n}] \cap W_{n,a_n}$.

The family of continuous functions $\{f_{n,a_n}\}$ chosen above has properties:

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