

191. Boltzmann Equation on Some Algebraic Structure Concerning Struggle for Existence

By Yoshiaki ITOH

Department of Mathematics, Tokyo Institute of Technology

(Comm. by Kinjirô KUNUGI, M. J. A., May 12, 1971)

V. Volterra had treated the problem of struggle for existence in his book based on the biological interest.

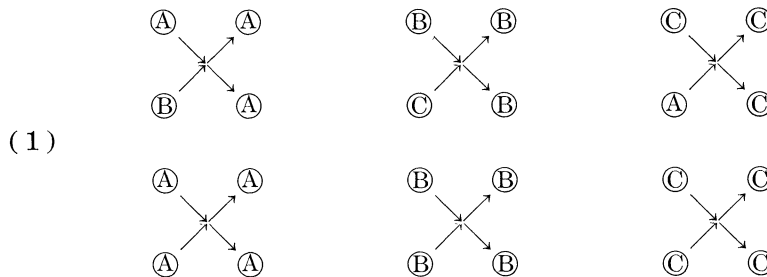
Here we will consider some models of struggle for existence. Although the equations are essentially the same to the Volterra's case, the formulations is different from it. The mathematical formulation is done analogously to the case of Kac's caricature of a Maxwellian gas. The Boltzmann equation on some finite algebraic structure will be mentioned.

Model I.

1) There are three types of particles, A , B and C , whose numbers are respectively n_A , n_B and n_C . $n_A + n_B + n_C = n$ is a constant integer.

2) Each particle is in a chaotic bath of like particles. Uniform distribution of colliding pairs is assumed.

3) Particles vary by the following collision rule:



that is to say, A is stronger than B , B is stronger than C and C is stronger than A .

Mathematical formulation. On the analogy of Boltzmann's problem, we may have

$$(2) \quad \begin{aligned} \frac{\partial}{\partial t} n_A &= \frac{n_A}{n} n_A + \frac{n_B}{n} n_A + \frac{n_A}{n} n_B - n_A \\ \frac{\partial}{\partial t} n_B &= \frac{n_B}{n} n_B + \frac{n_C}{n} n_B + \frac{n_B}{n} n_C - n_B \\ \frac{\partial}{\partial t} n_C &= \frac{n_C}{n} n_C + \frac{n_A}{n} n_C + \frac{n_C}{n} n_A - n_C. \end{aligned}$$

(2) is rewritten in the following way for sufficiently large n .