190. A Note on Ribbon 2-Knots

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1. We shall consider the 2-spheres in a 4-sphere that are locally flat, which will be called 2-*knots*. S. Kinoshita [2] showed that for each polynomial $f(t)$ with $f(1)=\pm 1$, there exists a 2-sphere in a 4-sphere whose Alexander polynomial is defined and equal to $f(t)$. Recently, by an another method, D. W. Sumners [4] [5] showed that the existence of the 2-knot K^2 such that i) the Alexander polynomial of K^2 is $f(t)$ above, and moreover, ii) the second homotopy group of the complement of K^2 has the "Γ-torsion".

It is easy to see that the 2-knots which S. Kinoshita constructed in [2] are ribbon 2-knots [6] [7]. He gave us the following question.

"Is every Sumners's 2-knot a ribbon 2-knot?"

In this paper we will give the affirmative answer of this question. We will consider everything from the combinatorial standpoint of view. By S^n, $\mathring{X}$, ∂X and $N(X, Y)$, we shall denote an n-sphere, the interior of X, the boundary of X and the regular neighborhood of X in Y, respectively. $X \simeq Y$ means that X is homeomorphic to Y, and $\overset{m}{\#} X$ the connected sum of the m copies of X.

2. We will give some knowledge of ribbon and Sumners's 2-knots [5] [7].

Definition 2.1. A locally flat 2-sphere K^2 in S^4 will be called a *ribbon* 2-knot, if there is a ribbon map ρ of a 3-ball B^3 into S^4 satisfying the following conditions

(1) $\rho \mid \partial B^3$ is an embedding and $\rho(\partial B^3)=K^2$,

(2) the self-intersections of B^3 by ρ consists of mutually disjoint 2-balls $D_1^2, \cdots, D_s^2$,

(3) the inverse set $\rho^{-1}(D_i^2)$ consists of disjoint 2-balls $D_i'^2$ and $D_i''^2$ such that $D_i'^2 \subset \mathring{B}^3$ and $\partial D_i''^2 = D_i''^2 \cap \partial B^3$ $(i=1, \cdots, s)$.

Let N_i^3 be a spherical-shell, which is homeomorphic to $S^2 \times [0,1]$ $(i=1, \cdots, m)$. A system of spherical-shells $N_1^3 \cup \cdots \cup N_m^3$ will be called *trivial* if they are mutually disjoint and such that

i) the 2-link $\partial N_1^3 \cup \cdots \cup \partial N_m^3$ of $2m$ components is of trivial type in $S^4-(\mathring{N}_1^3 \cup \cdots \cup \mathring{N}_m^3)$; that is, there are mutually disjoint 3-balls $B_1^3, \cdots, B_{2m}^3$ in $S^4-(\mathring{N}_1^3 \cup \cdots \cup \mathring{N}_m^3)$ such that $\partial N_i^3 = \partial B_i^3 \cup \partial B_{m+i}^3$ $(i=1, \cdots, m)$,

ii) for each i the 3-sphere $B_i^3 \cup N_i^3 \cup B_{m+i}^3$ bounds a 4-ball B_i^4 in S^4 such that $B_i^4 \cap B_j^4 = \emptyset$ $(i \neq j)$.