184. *Unimodular Numerical Contractions in Hilbert Space*

By I. ISTRĂŢESCU
Politechnic Institute Timişoare, Roumania

(Comm. by Kinjirô KUNUGI, M. J. A., May 12, 1971)

1. Let T be a unitary operator on a Hilbert space H. Then in particular

(i) T is a numerical contraction, i.e., $w(T) \leq 1$

(ii) the spectrum of T is a subset of the unit circle, i.e., $\sigma(T) \subset \{z, |z|=1\}$.

Call an arbitrary operator T a unimodular numerical contraction (u.n.c.) if it satisfies conditions (i) and (ii) above. Then similars to problems considered by B. Russo for unimodular contractions come to mind. The present paper has the aim to give some results in this direction.

2. In this section we give some easy results on unimodular numerical contractions concerning the eigenvalues.

Theorem 2.1. *If T is a unimodular numerical contration, then the following assertions hold:*

(i) *the eigenvectors of T corresponding to distinct eigenvalues are orthogonal,*

(ii) *if the eigenvectors of T span H, then T is unitary.*

Proof. For every $\xi \in \sigma(T)$ let $E_\xi(T)=\{x, Tx=\xi x\}$ and thus if T is a unimodular numerical contraction then

$$\xi \in \sigma(T) \cap \partial W(T)$$

and by Theorem 2 in [4], ξ is a normal eigenvalue, i.e.,

$$E_\xi(T)=E_{\xi^*}(T^*).$$

From this (i) follows immediately.

The assertion (ii) follows from (i) as in the case of unimodular contractions.

For the following result we need the notion of a maximal-single-valued extension of $R(z, x)=(T-z)^{-1}x$, $z \in \rho(T)=C_{\sigma(T)}$ and the following

Difinition 2.1. If T is a unimodular numerical contraction such that there does not exist an invariant subspace of T on which T is normal we call T completely unimodular numerical contraction.

Theorem 2.2. *Every completely unimodular numerical contraction has the property of maximal single-valued extension.*

Proof. If no, we find a number $\xi \in \sigma(T)$ and a non zero element $x \in H$ such that $Tx=\xi x$ and thus

$$E_\xi(T) \neq \{0\}$$

and $T|_{E_\xi(T)}$ is normal; a contradiction.